Cooperation in Two Person Games, Revisited

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Luminaries such as Nash (1953), Raiffa (1953), and Selten (1960), studied cooperation in two-person strategic games. We point out that when players may make monetary side payments, i.e., bimatrix games with *transferable utility* (TU), all previous solutions coincide. This solution is justified by simple axioms and an elementary decomposition of every such game into a (competitive) zero-sum game and a (cooperative) team game.

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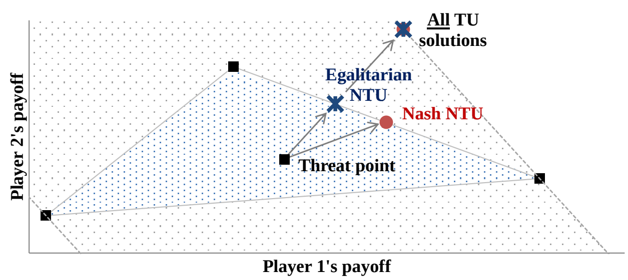
1. INTRODUCTION

Experiments and experience suggest that agreements may help resolve the tension between cooperation and competition observed in many games. Agreements are especially useful when monetary side payments are possible. We motivate with the simple bimatrix game below, representing the payoffs of two hot-dog vendors in a beach town. Every day, each player must decide whether to locate their cart at the airport (A), where there are 80 customers, or beach (B), where there are 200 customers. Say Player 1 profits \$3 per customer while Player 2 profits \$1 per customer. If the two collocate, then they divide the customers equally.

	A (airport)	B (beach)
A	120, 40	240, 200
B	600, 80	**300, 100**



The noncooperative dominant strategies of (B,B) yield total payoff of \$400, far short of the possible \$680. As is standard with *transferable utility* (TU), we assume that utility is measured in monetary terms, $u(\$x) = x$. We first review the *non-TU* (NTU) setting, i.e., with binding agreements but no side payments. In both bargaining and arbitration models discussed below, a pair of "hypothetical" (possibly randomized) strategies is considered, called threats for reasons to become clear soon. Let the resulting pair of (expected) payoffs be $t \in \mathbb{R}^2$. In the above example, if both players threaten B then $t = (300, 100)$. Now, it is only natural to choose a Pareto optimal outcome that improves both payoffs. Joint randomization may achieve any point in the convex hull of payoff pairs – the above triangle.

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NTU Bargaining. Nash's *bargaining problem* [1950] answers exactly this question: what should two parties do when faced with a joint decision involving a compact convex set $S \subset \mathbb{R}^2$ of possible payoff pairs, along with a threat point t which is achieved if the players fail to agree? Nash's solution is the point $u \geq t$ in S maximizing the product $(u_1 - t_1)(u_2 - t_2)$. Strategic and axiomatic justifications have been given for this solution. In our hot-dog example, the set S is the convex hull of payoff pairs. Nash's solution here gives (420,140), with players randomizing 50/50 between playing AB and BA. Kalai and Smorodinsky [1975] criticize Nash's solution for lacking monotonicity, and Kalai [1977] considers the "egalitarian" solution maximizing $\min\{u_1 - t_1, u_2 - t_2\}$.

Nash's [1953] "variable-threat bargaining" combines his noncooperative and cooperative solutions, as follows. He considers the game in which players first choose hypothetical mixed strategies to determine the threat point, and then assume they apply his bargaining solution. Of course, strategic players would choose threats to maximize the resulting bargaining solution. This threat selection game has unique Nash equilibrium payoffs – in our example it so happens to coincide with the bimatrix game's unique Nash equilibrium but this is not necessarily true in general.

NTU Arbitration. In independent work, Raiffa [1953] describes an arbiter for bimatrix games who, similarly, considers hypothetical mixed strategies for the two players and subsequently improves them to the Pareto frontier. Arbitration is appealing in that it is natural for an arbiter to seek an "equitable" outcome based upon the underlying strategic considerations, whereas bargaining may be subject to different negotiating abilities of the two players. The two are related since, in many negotiations, arbitration may serve as a looming threat or as a "fair" lower-bound on the players expectations. In summary, it is clear from the literature on bargaining and arbitration that there are a multitude of different NTU solutions.

2. DECOMPOSITION AND VALUE OF A TU BIMATRIX GAME

When one considers the possibility of side payments, the set of achievable payoffs trivially becomes a band between two lines of slope -1, as earlier illustrated. In such cases, all major bargaining and arbitration solutions coincide. Improving, relative to any fixed threat, amounts to the largest possible *equal* increase in payoffs.

Due to this concordance of theories, this payoff pair may be called "the value" of a TU bimatrix game. We [2010] present this as a generalization of the minmax value of a zero-sum game through the following decomposition. For any payoff matrices, $A, B \in \mathbb{R}^{m \times n}$:

$$(A, B) = \left(\frac{A-B}{2}, \frac{B-A}{2}\right) + \left(\frac{A+B}{2}, \frac{A+B}{2}\right).$$

In words, every game can be uniquely written as the sum of a *competitive* zero-sum game and a *cooperative* team game, where the players have identical payoffs. The value is the sum of the min-max value of the zero-sum game and the maximum value pair (the "max-max" value) of the team game. This cooperative-competitive "coco" decomposition can be used to compute the value in polynomial time, since valuing both zero-sum and team games is computationally efficient.

In our example, the value is $(\mathbf{440}, \mathbf{240}) = (100, -100) + (340, 340)$, as seen below:

$$\begin{pmatrix} 120,40 & 240,200 \\ 600,80 & 300,100 \end{pmatrix} = \begin{pmatrix} 40,-40 & 20,-20 \\ 260,-260 & \mathbf{100},-\mathbf{100} \end{pmatrix} + \begin{pmatrix} 80,80 & 220,220 \\ \mathbf{340},\mathbf{340} & 200,200 \end{pmatrix}.$$

3. AXIOMATIZATION

The Kalais also show that the value is the only solution satisfying the following five axioms on the bimatrix game, simpler but in the same spirit as Selten's [1960] axiomatization of the same value for two-player TU extensive forms.

1. **Pareto optimality.** The sum of the values is maximal.
2. **Payoff dominance.** If one player's payoff is larger than the opponent's in every cell, then that player's value must be larger.
3. **Shift invariance.** If the players' payoffs are shifted by the same additive constants (c_1, c_2) in every cell, then the value is shifted by the same constants.
4. **Redundant mixed strategies.** Adding a new strategy, which is equivalent to a mixture of two existing strategies (e.g., a new row which is a convex combination of two existing rows) does not change the value.
5. **Monotonicty in strategies.** If a player is given an additional strategy (e.g., a new row is added for player 1) then that player's value cannot decrease.

These properties are easily verified for zero-sum games and team games. In a noncooperative game, the last axiom, monotonicity is well-known to be violated. However, it is natural for an arbiter to reward (or at least not to punish) players for exposing more options, in the sense that it maximizes the total achieved.

4. CONCLUSIONS AND FUTURE WORK

As we have argued, existing cooperative theories favor a single value for TU bimatrix games. The solution's applicability is limited by several factors. First, it requires the ability for the players to make binding agreements with side payments. Second, it is essentially a solution in terms of payoffs (i.e., risk-neutral players), rather than permitting more general utilities. Third, no general solution concept is appropriate for all situations, e.g., Nash equilibrium has been repeatedly shown to be violated, and even the minmax value is inappropriate for some zero-sum games (e.g., computationally intractable games such as chess). Nonetheless, such theories shed light on strategic interaction. The *unique* and *efficiently computable* payoffs, unlike those of the noncooperative Nash equilibrium, are appealing.

We [2010] show how the above solution may be extended to address incomplete information, and they implement their first-best efficient outcome in restricted but important classes of Bayesian games. Incomplete information has proven to be extremely important in the study of noncooperative games but has received less attention in cooperative work.

Future work may empirically test these theories' predictions on face-to-face play of bimatrix games and study specific applications where cooperation is natural.

REFERENCES

Kalai, A., and E. Kalai 2010. Cooperation and competition in strategic games with private information. In *Proceedings 11th ACM Conference on Electronic Commerce* (EC), 345–346.

Kalai, E. 1977. Proportional solutions to bargaining situations: Interpersonal utility comparisons," *Econometrica* **45**:1623-1630.

Kalai, E. and M. Smorodinsky 1975. Other solutions to Nash's bargaining problem, *Econometrica* **43**:513–518.

Nash, J. F. 1950. The bargaining problem, *Econometrica* **18**:155–162.

Nash, J. F. 1953. Two-person cooperative games, *Econometrica,* **21**:128–140.

Raiffa, H. 1953. Arbitration schemes for generalized two-person games. In *Contributions to the Theory of Games II,* H. Kuhn, and A.W. Tucker, editors, 361–387.

Selten, R. 1960. Bewertung Strategischer Spiele, *Zeitschrift fur die gesamte Staatswissenschaft*, **116**(2):221–282.