# Influence diffusion dynamics and influence maximization in complex social networks



#### (Social) networks are natural phenomena



#### **Booming of online social networks**



Harvard, Oct. 18, 2011

# Opportunities and challenges on the research of online social networks

- Opportunities
  - massive data set, real time, dynamic, open
  - help social scientists to understand social interactions in a large scale
  - help marketing people to target to the right audience
  - help economists to understand social economic networks
- Challenges
  - graph structure based large scale data analysis
  - scalable graph algorithm design
  - realistic modeling of network formation, evolution, and information/influence diffusion in networks

# Our recent work on social network related research

- Social influence in social networks
  - scalable influence maximization
  - influence maximization with complex social interactions
- Game-theoretic based modeling of social interaction
  - bounded budget betweenness centrality game for network formation
  - Optimal pricing in social networks with networked effect
- Fundamental algorithms for large graphs
  - fast distance queries in power-law graphs
  - game-theoretic approach to community detection

## Scalable Influence Maximization in Social Networks

[KDD'09, KDD'10, ICDM'10] Collaborators: Chi Wang, Yajun Wang, Siyu Yang, Yifei Yuan, Li Zhang



- Word-of-mouth effect is believed to be a promising advertising strategy.
- Increasing popularity of online social networks may enable large scale WoM marketing

#### WoM (or Viral) Marketing

level of trust on different types of ads \*



\*source from Forrester Research and Intelliseek

# Two key components for studying WoM marketing

- Modeling influence diffusion dynamics, prior work includes:
  - independent cascade (IC) model
  - linear threshold (LT) model
  - voter model
- Influence maximization, prior work includes:
  - greedy approximation algorithm
  - centrality based heuristics

#### The Problem of Influence Maximization

- Social influence graph
  - vertices are individuals
  - links are social relationships
  - number p(u,v) on a directed link from u to v is the probability that v is activated by u after u is activated
- Independent cascade model
  - initially some seed nodes are activated
  - At each step, each newly activated node u activates its neighbor v with probability p(u,v)
- Influence maximization:
  - find k seeds that generate the largest expected influence



#### **Prior Work**

- Influence maximization as a discrete optimization problem proposed by Kempe, Kleinberg, and Tardos, 2003
  - Introduce Independent Cascade (IC) and Linear Threshold (LT) models
  - Finding optimal solution is provably hard (NP-hard)
  - Greedy approximation algorithm, 63% approximation of the optimal solution
    - select k seeds in k iterations
    - in each iteration, select one seed that provides the largest marginal increase in influence spread
- Several subsequent studies improved the running time
- Serious drawback:
  - very slow, not scalable: > 3 hrs on a 30k node graph for
    50 seeds

#### Our Work

- Exact influence computation is #P hard, for both IC and LT models --- computation bottleneck
- Design new heuristics
  - MIA (maximum influence arborescence) heuristic [KDD'10]
    - for general independent cascade model (more realistic)
    - 10<sup>3</sup> speedup --- from hours to seconds
    - influence spread close to that of the greedy algorithm of [KKT'03]
  - Degree discount heuristic [KDD'09]
    - for uniform independent cascade model
    - 10<sup>6</sup> speedup --- from hours to milliseconds
  - LDAG (local directed acyclic graph) heuristic [ICDM'10]
    - for the linear threshold model
    - 10<sup>3</sup> speedup --- from hours to seconds

#### Maximum Influence Arborescence (MIA) Heuristic

- For any pair of nodes u and v, find the maximum influence path (MIP) from u to v
- ignore MIPs with too small
  probabilities ( < parameter θ)</li>



#### MIA Heuristic (cont'd)

- Local influence regions
  - for every node v, all MIPs to v form its maximum influence in-arborescence (MIIA)



#### MIA Heuristic (cont'd)

- Local influence regions
  - for every node v, all MIPs to v form its maximum influence in-arborescence (MIIA)
  - for every node u, all MIPs from u form its maximum influence outarborescence (MIOA)
  - computing MIAs and the influence through MIAs is fast



# MIA Heuristic III: Computing Influence through the MIA structure

Recursive computation of activation probability ap(u) of a node u in its in-arborescence, given a seed set S

```
Algorithm 2 ap(u, S, MIIA(v, \theta))1: if u \in S then2: ap(u) = 13: else if Ch(u) = \emptyset then4: ap(u) = 05: else6: ap(u) = 1 - \prod_{w \in Ch(u)} (1 - ap(w) \cdot pp(w, u))7: end if
```

Can be used in the greedy algorithm for selecting k seeds, but not efficient enough

# MIA Heuristic IV: Efficient updates on incremental activation probabilities

- u is the new seed in MIIA(v)
- Naive update: for each candidate w, redo the computation in the previous page to compute w's incremental influence to v
  - $O(|MIIA(v)|^2)$
- Fast update: based on linear relationship of activation probabilities between any node w and root v, update incremental influence of all w's to v in two passes
  - O(|MIIA(v)|)



#### MIA Heuristic (cont'd)

- Iteration between two steps
  - Selecting the node v giving the largest marginal influence
  - Update MIAs after selecting v as the seed
- Key features:
  - updates are local
  - Iocal updates are linear to the local tree structure

#### **Experiment Results on MIA heuristic**

Influence spread vs. seed set size

running time



Experiment setup:

- 35k nodes from coauthorship graph in physics archive
- influence probability to a node v = 1 / (# of neighbors of v)
- running time is for selecting 50 seeds

#### Scalability of MIA heuristic



Experiment setup:

- synthesized graphs of different sizes generated from power-law graph model
- influence probability to a node v = 1 / (# of neighbors of v)
- running time is for selecting 50 seeds

#### Summary

- Scalable influence maximization algorithms
  - MixedGreedy and DegreeDiscount [KDD'09]
  - PMIA for the IC model [KDD'10]
  - LDAG for the LT model [ICDM'10]
- PMIA/LDAG have become state-of-the-art benchmark algorithms for Inf. Max.
- Collective citation count above 110 in less than 2 years

#### Handling Complex Social Interactions

[SDM'11, others under submissions] Alex Collins, Rachel Cummings, Te Ke, Zhenming Liu, David Rincon, Xiaorui Sun, Yajun Wang, Wei Wei, Yifei Yuan, Xinran He, Guojie Song, Yanhua Li, Katie Everett, Zhi-Li Zhang

## Handling complex social interactions

- people may dislike a product after usage and spread bad words about it
- a competing product may compete for social influence in the social network
- social relationships may be friends or foes

#### **Our solutions**

- people may dislike a product after usage and spread bad words about it
  - IC-N model and MIA-N algorithm
- a competing product may compete for social influence in the social network
  - CLT model and CLDAG algorithm for influence blocking maximization
- social relationships may be friends or foes
  - voter model in signed networks with exact inf. max. algorithm

#### IC-N model and MIA-N algorithm for the emergence and propagation of negative opinions

Harvard, Oct. 18, 2011



### Negative opinion model

- Extention of the independent cascade model
- The quality of the product to be advertised is characterized by the quality factor (QF)  $q \in [0,1]$ .
- Each node could be in 3 states
  - Inactive, positive, and negative.
- When node v becomes active,
  - If the influencer is negative, the activated influencee is also negative (negative node generates negative opinions).
  - If the influencer is positive, the activated influencee
    - is positive with prob. q.
    - is negative with prob. 1 q.
  - If multiple activations of a node occur at the same step, randomly pick one
  - Asymmetric --- negativity bias

#### Independent Cascading Process (without considering QF)



Independent Cascading Process (when considering QF)



# Our results (1)

• Complexity and approximation algorithm results

Scenario	<b>Objective function</b>	Algorithm result	Negative result
General directed graphs	Maximize expected positive nodes	$(1 - \frac{1}{e} - \varepsilon)$ -approx alg, due to submodularity	Exact sol. is NP hard.
General directed graphs	Maximize expected (positive – negative) nodes.	Exists an $(1 - \frac{1}{e} - \varepsilon)$ - approx alg. Only when q is sufficiently large	Same as above
Directed graphs with different <i>q</i> for different people	Maximize expected positive nodes	NA	Objective is non- submodular

### Our results (2)

• Q: is the knowledge of quality factor important?

- guess a "universally good" value q so that regardless of the actual quality factor, the seeds are good?
- No:  $\exists$  social networks s.t. a wrong guess of q could lead to a much worse result than the optimal one. ( $\Theta(\sqrt{n/k})$ )
- Intuition: which one seed to select in the following graph?



## Our results (3)

- Q: what is the bottleneck of the approx. alg.
  - Given a specific seed set S, can we evaluate the expected number of positive nodes?
    - In general, #P-hard; can use Monte Carlo to approximate.
    - But exists efficient exact algorithm for arborescence (trees).
  - Developed scalable heuristic MIA-N based on influence calculation alg. for arborescences.

# Computation in directed trees (in-arborescences)

Without negative opinions, a simple recursion computes the activation probability of *u*:

$$- ap(u) = 1 - \prod_{w \in N^{in}(u)} (1 - ap(w)p(w, u))$$

 Difficulty with negative opinions: needs to know whether the neighbors of *u* is positive or negative --- because of negativity bias



### Solutions for in-arborescences

 Step 1: compute activation probability of u at step t (via dynamic programming):

 $\begin{array}{ll} ap(u,t) = & t = 0 \land u \in S, \\ 1 & t = 0 \land u \notin S, \\ 0 & t = 0 \land u \notin S, \\ 0 & t > 0 \land u \notin S, \\ \prod_{w \in N^{in}(u)} [1 - \sum_{i=0}^{t-2} ap(w,i)p(w,u)] & t > 0 \land u \notin S. \\ -\prod_{w \in N^{in}(u)} [1 - \sum_{i=0}^{t-1} ap(w,i)p(w,u)] & t > 0 \land u \notin S. \end{array}$ 

 Step 2: compute positive activation probability of u at step t:

$$pap(u,t) = ap(u,t) \cdot q^{t+1}.$$

#### Influence spread and QF



- Results on a collaboration network with 15K nodes.
- Convex function because of negativity bias

#### Performance of the heuristic



• MIA-N heuristic performs nearly as good as the original greedy algorithm.

#### Scalability



• MIA-N heuristic is 3 orders of magnitude faster than Greedy

CLT model for competitive influence diffusion and CLDAG algorithm for the influence blocking maximization problem

## The problem

- Consider two competing influence diffusion process, one positive and one negative
- Inf. Blocking Max.: selecting positive seeds to block the negative influence diffusion as much as possible
  - e.g. stop rumors on a company, on a political candidate, on public safety events, etc.

#### **Our solution**

- Competitive linear threshold model
  - positive influence and negative influence diffuse concurrently in the network
  - negative influence dominates in direct competition
- Prove that the objective function is submodular
- Design scalable algorithm CLDAG to achieve fast blocking effect

# Influence diffusion on networks with friends and foes

Harvard, Oct. 18, 2011

## The problem

- You would positively influence your friends, but influence your foes in the reverse direction
- How to model such influence?
- How to design influence maximization algorithm?

#### **Our solution**

- Voter model in signed networks
  - suitable for opinion changes from positive to negative or reverse
  - individual takes the opposite opinion from his foe
- Provide complete characterization of short term dynamics and long-term steady state behavior
- Provide exact solutions to the influence maximization problem

## On going and future directions

- Model validation and influence analysis from real data
- Even faster heuristic algorithms
- Fast approximate algorithms
- Online and adaptive algorithms

#### Questions?

Harvard, Oct. 18, 2011