The enigmatic 12/5 fractional quantum Hall effect

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We numerically study the fractional quantum Hall effect at filling factors $\nu = 12/5$ and 13/5 (the particle-hole conjugate of 12/5) in high-quality two-dimensional GaAs heterostructures via exact diagonalization including finite well width and Landau level mixing. We find that Landau level mixing suppresses $\nu = 13/5$ fractional quantum Hall effect relative to $\nu = 12/5$. By contrast, we find both $\nu = 2/5$ and (its particle-hole conjugate) $\nu = 3/5$ fractional quantum Hall effects in the lowest Landau level to be robust under Landau level mixing and finite well-width corrections. Our results provide a possible explanation for the experimental absence of the 13/5 fractional quantum Hall state as caused by Landau level mixing effects.

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Introduction – There is interest across physics, mathematics, engineering, materials research, and computer science in finding robust experimental manifestations of topologically ordered phases with non-Abelian anyonic low-energy excitations. Not only are non-Abelian anyons (i.e., neither fermions nor bosons) suitable for topological quantum computation, but they are described by topological quantum field theories (TQFT) of various universality classes of intrinsic fundamental interest [1]. The fractional quantum Hall effect [2–4] (FQHE) is the canonical example of a system supporting topologically ordered phases and is widely thought to support non-Abelian anyons in the second orbital electronic Landau level (LL), most probably at filling factor $\nu = 5/2$ [5]. There is a possibility that the experimentally observed FQHE at $\nu = 12/5$ supports particularly exotic topologically ordered phases described by the Z_k parafermionic Read-Rezayi states [6–13], exemplifying an exotic $\mathrm{SU}(2)_3$ TQFT (in contrast to the 5/2 FQH state belonging to the $\mathrm{SU}(2)_2$ TQFT). Since $\mathrm{SU}(2)_3$ TQFT supports a richer version of non-Abelian anyons that can realize *universal* fault-tolerant quantum computation [1], there is a great deal of interest in the 12/5 FQHE. In this work, we focus on the enigmatic FQHE at $\nu = 12/5$ that has attracted considerable recent experimental and theoretical attention.

Compared to the rather ubiquitous $\nu = 5/2$ FQHE, the experimental literature for $\nu = 12/5$ (= 2 + 2/5 filling) is sparse with only about five experimental reports of its observation. The 12/5 FQHE was observed in a 30 nm wide GaAs quantum well with electron densities of $n \sim 3 \times 10^{11}\mathrm{cm}^{-2}$ at magnetic field strengths of $B \sim 5$ Tesla at temperatures $T \sim$ 6-36 mK [14–18]. In addition to its fragility (the 12/5 FQHE is observed only in the highest quality samples with little disorder), the real enigma is the corresponding particle-hole conjugate FQHE at 13/5 (= $5 - 12/5$) has never been observed in spite of other FQHE in the second LL (e.g., 7/3 and 8/3, 11/5 and 14/5) showing both particle-hole conjugate states with roughly equal strength. This discrepancy is puzzling because in the lowest LL the FQHE at $\nu = 2/5$ and 3/5 are both routinely observed, are to good approximation particle-hole conjugates of one another [19, 20], and are well-described by the composite fermion (CF) theory [4, 21]. More mysterious (and theoretically interesting) is that the 12/5 and 13/5 FQHE (with roughly equal strength) are observed in systems where two subbands are occupied (e.g., bilayers, thick quantum wells) such that the chemical potential is in the lowest LL (but in the higher subband so two LLs are still completely full) [22–24]. In this work we provide a possible explanation for the absence (presence) of 13/5 (12/5) FQHE in the second LL as arising from the LL mixing effect that explicitly breaks the particle-hole symmetry.

Several candidate wave functions for $\nu = 12/5$ have been proposed and studied [8–10] under idealized conditions, using the Coulomb interaction without particle-hole symmetry breaking. Two recent numerical studies [9, 10] reinforced initial results [6, 7] that the ground state at $\nu = 12/5$ is in the non-Abelian Z_3 Read-Rezayi (RR) phase. Both studies perturbed the interaction finding a finite region of stability around the Coulomb point. All works considered particle-hole symmetric two-body Hamiltonians so all conclusions made therein regarding the $\nu = 12/5$ state are equally valid for the particle-hole conjugate state at $\nu = 13/5$. Thus, existing theories provide evidence that the experimentally observed 12/5 and (unobserved) 13/5 FQHE are both in the RR Z_3 phase but cannot explain at all why one (i.e., 12/5) exists experimentally and the other (i.e., 13/5) does not. We provide a plausible explanation for this puzzle.

LL mixing breaks particle-hole symmetry through emergent three-body (and higher) terms in an effective realistic Hamiltonian [25–27]. The importance of LL mixing can be parameterized by the ratio κ of the Coulomb energy $e^2/\epsilon l_0$ to the bare cyclotron energy $\hbar\omega$ (i.e., the LL gap): $\kappa = (e^2/\epsilon l_0)/\hbar\omega$ where ϵ is the background lattice dielectric constant, $l_0 = \sqrt{\hbar c/eB}$ is the magnetic length, e is the electron charge, and $\omega = eB/mc$ is the cyclotron frequency. For

GaAs, $\kappa \approx 2.5/\sqrt{B[\mathrm{T}]}$. For most experiments in the second LL, κ is of order unity, making LL mixing an important correction. One attempt at incorporating LL mixing at $\nu = 12/5$ used the approximation of including additional basis states within exact diagonalization [28], but did not investigate 13/5. In the present work, we numerically study a realistic model of the FQHE in the second LL using exact diagonalization, systematically including LL mixing effects due to (the infinite number of) all other LLs. We find that the LL mixing-induced particle-hole symmetry breaking strongly favors the $\nu = 12/5$ FQHE over the 13/5 in the second LL, qualitatively in agreement with experimental observations. By contrast, in the lowest LL we do not find significant particle-hole symmetry breaking between $\nu = 2/5$ and 3/5 FQHE. Our work explains the presence (absence) of 12/5 (13/5) in the second LL while at the same time explaining the existence and equal strength of 2/5 and 3/5 FQHE in the lowest LL. Our work also strengthens the claim that at finite LL mixing 12/5 FQHE arises from a RR parafermionic non-Abelian state (rather than from Abelian composite fermion states as for the 2/5 and 3/5 FQHE).

Effective Hamiltonian – Our realistic effective Hamiltonian describes N_e interacting electrons confined to the N^{th} LL of a quasi-two-dimensional quantum well (modelled as an infinitely deep square well) and incorporates LL and subband mixing. Finite width reduces the Coulomb interaction at short range and the Coulomb interaction causes virtual electron/hole excitations to higher/lower LLs and subbands included perturbatively to lowest order in κ (note this involves coupling all LLs [26]). The effective Hamiltonian is

$$H(w/\ell_0,\kappa,N) = \sum_m V^{(N)}_{2\text{body},m}(w/\ell_0,\kappa)\sum_{i<j}\hat{P}_{ij}(m) + \sum_m V^{(N)}_{3\text{body},m}(w/\ell_0,\kappa)\sum_{i<j<k}\hat{P}_{ijk}(m) \quad (1)$$

where $\hat{P}_{ij}(m)$ and $\hat{P}_{ijk}(m)$ are two- and three-body projection operators onto pairs or triplets of electrons with relative angular momentum m. $V^{(N)}_{2\text{body},m}(w/\ell_0,\kappa)$ and $V^{(N)}_{3\text{body},m}(w/\ell_0,\kappa)$ are the two- and three-body effective pseudopotentials [29, 30] in the N^{th} LL–we use planar pseudopotentials throughout this work. Beyond renormalizing the two-body interactions, LL mixing produces particle-hole symmetry breaking three-body terms (cf. Ref. 26). Eq. (1) has a well-defined exact limit as $\kappa \to 0$, hence, we can determine the leading order effects of LL mixing on the FQHE. Most experimental observations of the 12/5 FQHE occur at fields of $B \sim 5.15$ T (see Ref. 15) giving a quantum well width (30 nm) of $w/l_0 \approx 2.65$ and $\kappa \approx 1.1$. We estimate (an exact self-consistent calculation is possible for a particular device [31]) that an infinitely deep quantum well of $w/l_0 \approx 3$ provides approximately the same confinement as the real quantum well, and we consider $w/l_0 \leq 4$ and $\kappa \neq 0$ to model realistic samples under realistic conditions. Theory and experiment both suggest $\nu = 12/5$ (13/5) to be fully spin-polarized and we make that assumption throughout this work.

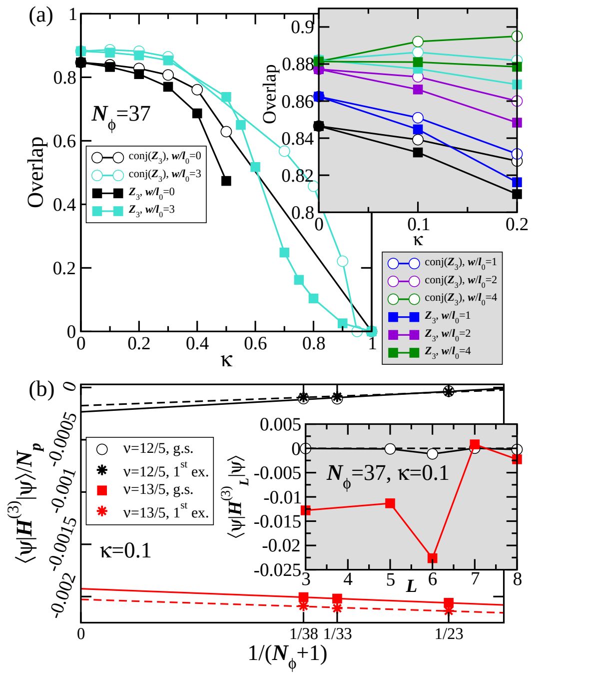

FIG. 1. (Color online) (a) Wave function overlap between Z_3 and conj(Z_3) and the exact ground state of Eq. (1) at $\nu = 13/5$ and 12/5, respectively, as a function of κ for $N_\phi = 37$ (14 holes/electrons). A finite well width increases the overlaps and κ breaks particle-hole symmetry yielding higher overlaps with conj(Z_3) for 12/5 compared to Z_3 for 13/5. The inset shows the overlaps in more detail. (b) Expectation values of the three-body terms per particle N_p of Eq. (1) for $\kappa = 0.1$ and $w/l_0 = 0$, evaluated for the ideal Coulomb ground and first excited states at 12/5 and 13/5, respectively, as a function of inverse LL degeneracy $[1/(N_\phi + 1)]$ extrapolated to the thermodynamic limit. $N_\phi = 27$ is aliased with $\nu = 1/3$ and left out. (Inset) Expectation values for each three-body term $[H^{(3)}_L = V^{(N)}_{3\text{body},L}(w/\ell_0,\kappa)\sum_{i<j<k}\hat{P}_{ijk}(L)]$ for $N_\phi = 37$. Lines are a guide to the eye except in the main plot of (b) where they represent linear extrapolations.

We consider $V^{(3)}_{3\text{body},m}$ for $3 \leq m \leq 8$–the psuedopotentials become matrix-valued for some m above $m > 9$ and previous work demonstrated that these terms are unlikely to produce qualitative effects [31], especially for small κ.

We use the spherical geometry [4, 29] where the total magnetic flux $N_\phi = N_e/f - S$ where f is the filling factor, as $N_e \to \infty$, of the N^{th} LL and S is the shift [32]. The experimentally filling factor $\nu = f + 2N$ where $2N$ arises from completely filling the lower N spin-up and down LLs. FQHE states are gapped uniform density ground states with total angular momentum $L = 0$. The RR Z_3 state describes $f = 3/5$ with $S = 3$ while the particle-hole conjugate RR state, conj(Z_3), describes $f = 2/5$ with $S = -2$. The CF states for $\nu = 2/5$ and 3/5 have shifts of $S = 4$ and -1, respectively.

Overlap, perturbation theory, and entanglement spectra– We first investigate whether the system remains in the Z_3 RR phase under realistic conditions. The ground state of Eq. (1)

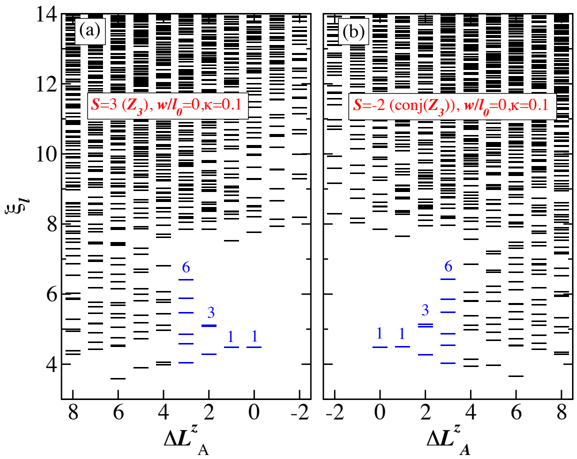

FIG. 2. (Color online) (a) Entanglement spectrum for the exact ground state of Eq. (1) for $w/l_0 = 3$ and $\kappa = 0.1$ at $\nu = 13/5$ (shift $S = 3$) and (b) at $\nu = 12/5$ (shift $S = -2$) for $N_\phi = 37$. The counting for the low-lying levels is 1, 1, 3, 6 up to $\Delta L^z_A = 5$ agreeing with Z_3 and conj(Z_3). The orbital cuts, using the notation of Ref. 33, are $P[0|0]$ for $S = 3$ and $P[1|1]$ for $S = -2$. $\Delta L^a_A = L^z_A - (L^z_A)_{\rm root}$ where (a) $(L^z_A)_{\rm root} = 120$ and (b) $(L^z_A)_{\rm root} = 60.5$.

is uniform with $L = 0$ for the RR shifts for all system sizes up to $N_\phi = 37$ for $\kappa \neq 0$ and $N_\phi = 42$ for $\kappa = 0$ (we have not studied $\kappa \neq 0$ for $N_\phi = 42$). The ground states have $L \neq 0$ for the CF shifts for zero and non-zero κ, for most system sizes. The Bonderson-Slingerland non-Abelian state for $\nu = 12/5$ [34] has $L = 0$ at $\kappa = 0$ but a smaller gap than the RR state [8]–this behavior remains with $\kappa \neq 0$. Similar qualitative results were recently found in the $\kappa = 0$ limit using the density matrix renormalization group [9, 10].

Fig. 1(a) presents the overlap between the exact ground state $|\psi\rangle$ of Eq. (1) with the model wave functions [Z_3 and conj(Z_3)]. For small κ the overlap remains relatively unchanged but the 12/5 overlap with conj(Z_3) is larger than the overlap with Z_3 at 13/5 for $\kappa \lesssim 0.5$ for all system sizes–the overlap at 13/5 decreases monotonically with κ and both overlaps are found to collapse to zero near $\kappa \approx 1$ though some finite size effects are observed for larger κ.

Since the overlaps are relatively flat for small κ, we study the eigenstates for $\kappa = 0$. We calculate $\langle\psi|H^{(3)}|\psi\rangle$ where $H^{(3)} = \sum_m V^{(N)}_{3{\rm body},m}(w/\ell_0, \kappa) \sum_{i<j<k} \hat{P}_{ijk}(m)$ [shown in Fig. 1(b)]–this represents the lowest-order perturbative contribution to particle-hole symmetry breaking induced by LL mixing. The thermodynamic limit extrapolation of $\langle\psi|H^{(3)}|\psi\rangle$ per particle for $\nu = 12/5$ at $\kappa = 0$ is more than ten times smaller than for 13/5, indicating that LL mixing more severely affects the energetics of 13/5 compared to 12/5. While the ground state energies are seemingly lowered by the three-body terms, the excited states are lowered as well, reducing the energy gap at 13/5 and increasing the gap at 12/5. In the inset of Fig. 1(b) we show that $V^{(3)}_3$, $V^{(3)}_5$, and $V^{(3)}_6$ are the three-body pseudopotentials that contribute most to

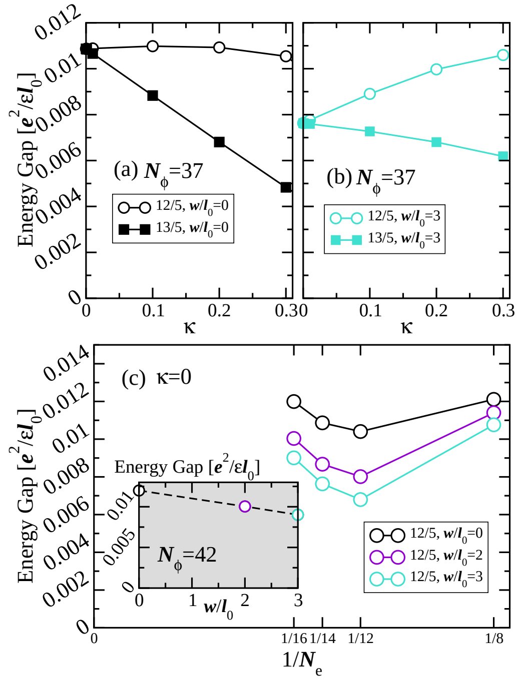

FIG. 3. (Color online) Energy gap for N_ϕ=37 at $\nu = 12/5$ and 13/5 for $w/l_0 = 0$ (a) and 3 (b). Similar results are obtained for smaller system sizes. (c) Width dependence of the gap for N_e =8, 12, 14, and 16 for $\nu = 12/5$ for $w/l_0 = 0$, 2, and 3 and $\kappa = 0$. (Inset) The gap as a function of w/l_0 at $\kappa = 0$ for $N_e = 16$ ($N_\phi = 42$). Finite width reduces the gaps by approximately 25% at $w/l_0 = 3$ relative $w/l_0 = 0$ for the largest system size. Note the similarities in (c) to Fig. 1(b) in Ref. 9.

particle-hole symmetry breaking between $\nu = 12/5$ and 13/5. The Z_3 state has a relative abundance of three-body clustering with low total angular momentum by construction [6] and large expectation values of $H^{(3)}_L$, similar to $|\psi\rangle$ for $\kappa = 0$ at $\nu = 13/5$. In contrast, the three-body terms have little effect on 12/5.

Overlaps may depend on short-range physics, so we investigate orbital entanglement spectra [33, 35–39]. If the ground state is in the RR phase, the counting of the low-lying levels of the entanglement spectra will be related to the SU$(2)_3$ TQFT describing the edge excitations [33]. The counting of the low-lying levels for $\nu = 13/5$ and 12/5 for $w/l_0 = 3$ and $\kappa = 0.1$ (Fig. 2) matches the counting for Z_3 and conj(Z_3), respectively, (including $\kappa = 0$, see Ref. 9).

The results above confirm that the ground state of Eq. (1) remains in the RR phase under LL mixing. Further, LL mixing affects $\nu = 13/5$ more than 12/5 and introduces strong particle-hole asymmetry.

Energy gap–The neutral gap is related to the experimentally measured activation gap and the physical robustness of the FQHE. It is the difference between the two lowest energies at constant N_ϕ, if the ground state has $L = 0$, otherwise it is taken to be zero. Fig. 3(a) and (b) show energy gaps for our largest system ($N_\phi = 37$) for $w/l_0 = 0$ and

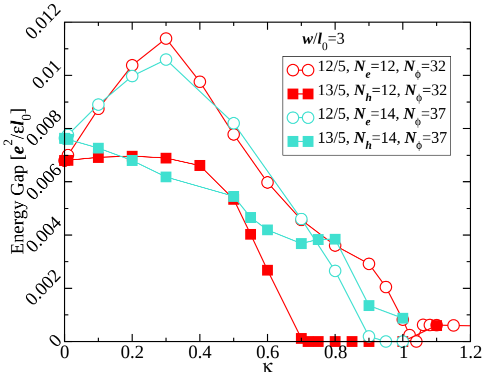

FIG. 4. (Color online) Energy gap for $\nu = 12/5$ and 13/5 as a function of κ for $w/l_0 = 3$ for $N_\phi = 32$ and 37. We note for $\kappa \gtrsim 0.6$ the gap behavior is no longer consistent between system sizes.

3 (see Supplementary Materials for other w/l_0). LL mixing breaks particle-hole symmetry producing a larger energy gap for $\nu = 12/5$ compared to 13/5. The gap at $w/l_0 = 3$ for 12/5 increases with κ while the 13/5 gap is suppressed (the suppression is found for all non-aliased system sizes and values of w/l_0, however an increasing gap at $\nu = 12/5$ for non-zero width is only found for the two largest system sizes $N_\phi = 37$ and 32). Hence, LL mixing strengthens the 12/5 FQHE for finite w/l_0 while weakening 13/5 (strengthening of the FQHE gap with LL mixing does not happen for $\nu = 5/2$ [31]).

The thermodynamic extrapolation suffers from finite-size effects ($N_\phi = 12$ and 17) and aliasing ($N_\phi = 27$). The energy gaps at the remaining N_ϕ are shown in Fig. 3(c). Without LL mixing, finite width decreases the gap from $0.012e^2/\epsilon l_0$ at $w/l_0 = 0$ to $0.009e^2/\epsilon l_0$ at $w/l_0 = 3$ [values given are for $N_\phi = 42$ shown in the inset of Fig. 3(c)]. In the limit of small LL mixing, (i.e., high magnetic fields) it should be possible to observe more robust 12/5 states in narrow quantum wells.

Fig. 4 shows the energy gap as a function of κ for $N_\phi = 32$ and 37 (12 and 14 electrons (holes) for $\nu = 12/5$ (13/5), respectively) to the experimental value of $\kappa \sim 1.1$ for $w/l_0 = 3$. All the sharp features in the κ-dependence are associated with the change of L in the first excited states. The behavior of the different system sizes is consistent up to $\kappa = 0.6 - 0.7$ and demonstrates a larger energy gap at 12/5 than at 13/5. Finite-size effects are observed for larger κ which could be a result of our perturbative (in κ) approach to LL mixing breaking down or the smallness of the energy gap.

Second versus lowest Landau level–Finally we compare the second with the lowest LL. In Fig. 5(a) we show the relative energy gap difference induced by LL mixing between $\nu = 12/5$ and 13/5 and between $\nu = 2/5$ and 3/5 as a function of particle number. The LL mixing induced difference is much larger in the second LL than in the lowest LL (the sign is also different between the two with 12/5 strongly favored in the second LL while 3/5 is slightly favored in the lowest LL). The LL mixing induced gap difference between 12/5 and 13/5 grows with system size and is likely a robust feature in

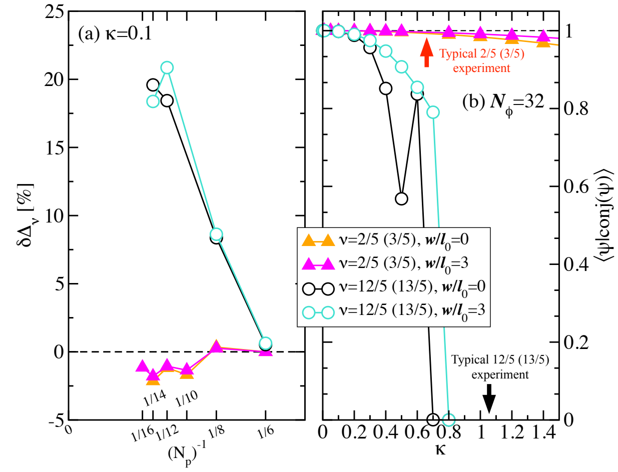

FIG. 5. (Color online) (a) Relative gap difference $\delta\Delta_\nu = (\Delta_\nu - \Delta_{1-\nu})/\Delta_\nu$ (induced by $\kappa = 0.1$) between particle-hole-conjugates at 12/5 (13/5) and 2/5 (3/5). N_p is the number of particles for $\nu = 12/5$ and 2/5 or number of holes for $\nu = 13/5$ and 3/5. (b) Particle-hole symmetry breaking (quantified by $\langle\psi|\text{conj}(\psi)\rangle$) in the second LL compared to the lowest LL for $w/l_0 = 0$ and 3. The system sizes are $N_\phi = 32$ for $\nu = 12/5$ (13/5) and $N_\phi = 31$ for $\nu = 2/5$ (3/5).

the thermodynamic limit.

We can further quantify the particle-hole symmetry breaking by calculating the overlap between the exact ground state $|\psi\rangle$ at $\nu = 12/5$ (2/5) and the particle-hole conjugate of the exact ground state $|\text{conj}(\psi)\rangle$ at $\nu = 13/5$ (3/5). At $\kappa = 0$, this overlap is unity since the two states are particle-hole conjugates. In Fig. 5(b) particle-hole symmetry is much more strongly broken for the $\nu = 12/5$ (13/5) FQHE than for the $\nu = 2/5$ (3/5) FQHE. In fact, particle-hole symmetry is hardly broken at all in the lowest LL (in the lowest LL $\langle\psi|\text{conj}(\psi)\rangle \gtrsim 0.9$ up to $\kappa \sim 2.4$). This apparent particle-hole symmetry could be a property of the lowest LL or of the CF-like states in any LL.

Conclusion – LL mixing strongly breaks the particle-hole symmetry between $\nu = 12/5$ and 13/5 FQHE in the second LL, but has little effect on $\nu = 2/5$ and 3/5 FQHE in the lowest LL. Our work implies that the absence of 13/5 FQHE in the second LL is a direct consequence of LL mixing effects. This is mainly due to the suppression of the energy gap at $\nu = 13/5$ – the FQHE might simply be too fragile (in terms of energy gap) since LL mixing affects 13/5 more severely than 12/5, and because in experimental measurements, at constant density, κ is larger at 13/5 compared to 12/5. The 12/5 ground state at shift $S = -2$ remains in the non-Abelian parafermionic (conjugate) RR Z_3 phase when finite-width and non-zero LL mixing are taken into account extending the validity of previous conclusions [6, 7, 9, 10, 28] obtained for idealized conditions. We do not rule out the $\nu = 13/5$ FQHE in the Z_3 RR phase, but establish that the 13/5 FQHE is always much weaker than 12/5. Future experiments with smaller κ could show a very weak FQHE at $\nu = 13/5$ in extremely high mobility samples at ultra-low temperatures with a very small activation energy.

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Supplementary Materials: The enigmatic 12/5 fractional quantum Hall effect

ENERGY GAPS FOR $w = 1, 2, 4$

In Fig. S1 we present the energy gaps calculated for $w/l_0 = 1, 2$, and 4. Together with Fig. 3 they cover the range of possible effective widths that could be realized in experiment. For all non-aliased system sizes and for all widths we observe that the gap of the 13/5 state is suppressed relative to the 12/5 state gap.

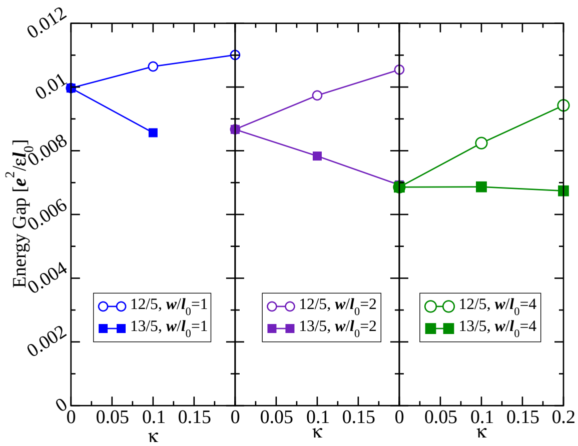

FIG. S1. (Color online) Energy gap (lowest two eigenvalues difference) for a system with N_ϕ=37 at 12/5 and 13/5.

We expect that the equivalence of various models of finite widths demonstrated for $\nu = 5/2$ [31] also holds here. Thus, to determine the effective width w corresponding to a certain experimental device one would first calculate (for instance using a Schrodinger-Poisson solver) or measure [40] the square of the absolute value of the electron wave function in the direction perpendicular to the 2DEG and determine its variance (as defined in Ref. [31]). Then w should be chosen such that the variance in the ground state of an infinitely deep quantum well of width w is the same as in the given experimental sample.

As can be seen from the Fig. S1 finite width significantly reduces the gap and thus the strength of the 12/5 state. Therefore an experiment in narrow quantum wells could obtain more robust 12/5 state provided that all the effects neglected in our model (such as quantum well interface scattering, etc.) remain unchanged.

TOPOLOGICAL GAP AT 12/5 AND 13/5

We define the topological gap following Ref. 33 as the difference between the lowest lying and the next state in the entanglement spectrum corresponding to $\Delta L^z_A = 1$ (as defined in the caption of Fig. 2). It represents the "energy difference" between the universal part of the entanglement spectrum, describing the [non-Abelian in case of RR and conj(RR)] modes and the generic continuum of states.

From the data presented in Fig. S2 we identify two trends: first, the topological gap increases with increased finite width, and second, Landau level mixing leads to the suppression of the topological gap at 13/5 relative to 12/5 in the same way it is observed for the energy gap, giving support to the main conclusion of this work based on a different measure.

ROBUSTNESS OF THE 2/5 AND 3/5 FQH STATES TO THE LANDAU LEVEL MIXING

To characterize the evolution of the states in the lowest Landau level we approximate the composite fermion states at 2/5 and 3/5 with the exact ground state of a "hardcore" model Hamiltonian with $V_1 \neq 0$ and all other $V_m = 0$ at the at $N_\phi = 5N_e/2 - 4$ and $N_\phi = 5N_e/3 + 1$, respectively. This Hamiltonian produces the $1/m$ Laughlin state exactly for $N_\phi = m(N_e - 1)$ and produces ground states with extremely large overlap (> 0.99) with composite fermion states with filling factor $\nu = n/(2pn+1)$ at the appropriate flux as checked via Monte Carlo. As shown in Fig. S3 the overlap remains quite stable under Landau level mixing and only starts to significantly decrease around $\kappa = 3 - 4$, well beyond the typical experimental values.

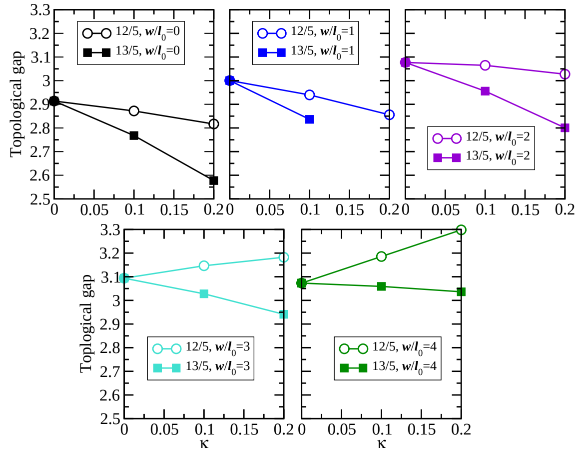

FIG. S2. (Color online) Topological gap for 12/5 and 13/5 for $N_\phi = 37$ and (top to bottom) $w = 0, 1, 2, 3, 4$.

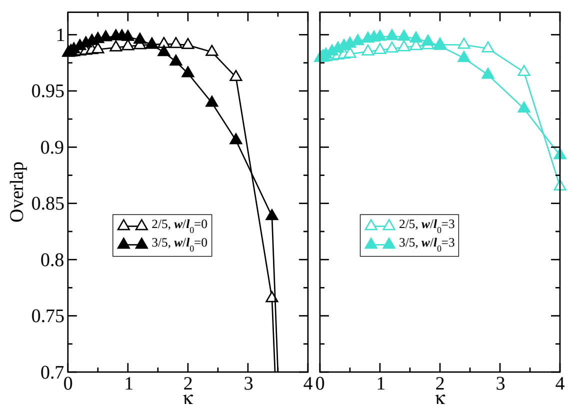

FIG. S3. (Color online) Overlap between the realistic ground state and the ground state of the hardcore ($V_1 \neq 0$ and $V_m = 0$ for all other m) Hamiltonian for $N_\phi = 31$. w=0 (left panel) and w=3 (right panel).

It is an open question whether the observed robustness of the FQH states at 2/5 and 3/5 is due to their CF-like nature or to the specific form of the effective interaction in the lowest Landau level.