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Combinatorial Online Learning

for Combinatorial Optimizations

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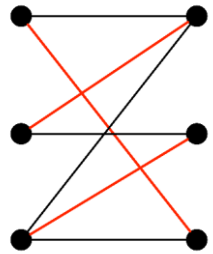
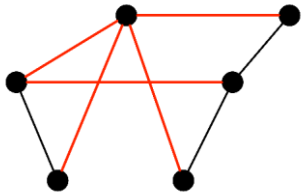
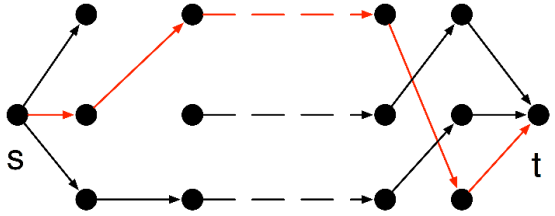
Combinatorial optimization

- Well studied
  - classics: max. matchings, shortest paths, min. spanning trees,
  - modern applications: online advertising, viral marketing
- What if the inputs are stochastic, unknown, and has to be learned over time?
  - link delays
  - click-through probabilities
  - influence probabilities in social networks





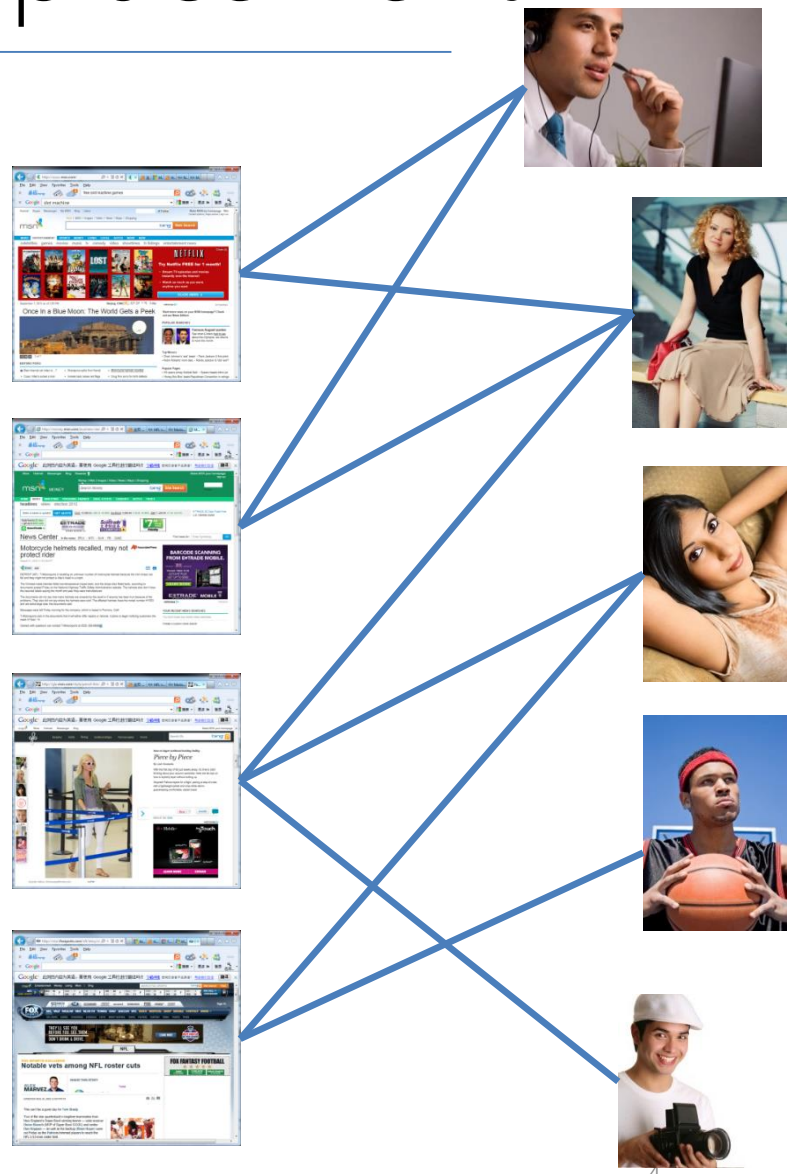


Combinatorial online learning for combinatorial optimizations

- Need new framework for learning and optimization:
- Learn inputs while doing optimization --- combinatorial online learning
- Learning inputs first (and fast) for subsequent optimization --- combinatorial pure exploration

Motivating application: Display ad placement

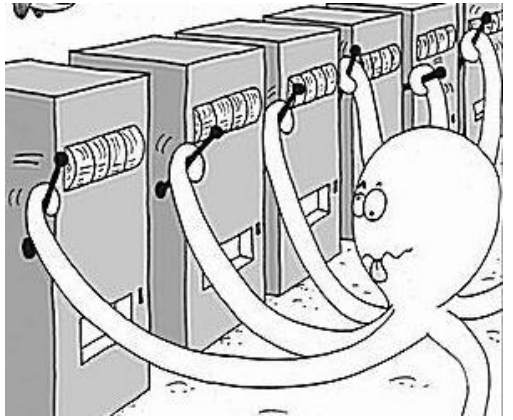
- Bipartite graph of pages and users who are interested in certain pages
  - Each edge has a click-through probability
- Find k pages to put ads to maximize total number of users clicking through the ad
- When click-through probabilities are known, can be solved by approximation
- Question: how to learn click-through prob. while doing optimization?



Multi-armed bandit: the canonical OL problem

- There are $\boldsymbol{m}$ arms (machines)
- Arm $\boldsymbol{i}$ has an unknown reward distribution with unknown mean $\boldsymbol{\mu_i}$
  - best arm $\boldsymbol{\mu^* = \max \mu_i}$
- In each round, the player selects one arm to play and observes the reward

Multi-armed bandit problem



- Regret after playing T rounds:
  - Regret $= T\mu^* - \mathbb{E}[\sum_{t=1}^{T} R_t(i_t^A)\ ]$
- Objective: minimize regret in T rounds
- Balancing exploitation-exploration tradeoff
- Known results:
  - UCB1 (Upper Confidence Bound) [Auer, Cesa-Bianchi, Fischer 2002]
    - Gap-dependent bound $\mathbf{O}(\log T \sum_{i:\Delta_i>0} 1/\Delta_i)$, $\Delta_i = \mu^* - \mu_i$, match lower bound
    - Gap-free bound $\mathbf{O}(\sqrt{mT \log T})$, tight up to a factor of $\sqrt{\log T}$

Naïve application of MAB

- Every set of k webpages is treated as an arm
- Reward of an arm is the total click-through counted by the number of people
- Issues
  - combinatorial explosion
  - ad-user click-through information is wasted

Issues when applying MAB to combinatorial setting

- The action space is exponential
  - Cannot even try each action once
- The offline optimization problem may already be hard
- The reward of a combinatorial action may not be linear on its components
- The reward may depend not only on the means of its component rewards

Combinatorial Online Learning

- General framework
  - Treat offline optimization as an oracle
  - Characterize general learning conditions
  - Focus on online tradeoff between exploration and exploitation
- My existing work
  - ICML'13/JMLR'16: general combinatorial multi-armed bandit (CMAB) framework, apply to non-linear rewards, approximation oracle, probabilistically triggered arms
  - ICML'14: combinatorial partial monitoring
  - NIPS'14: combinatorial pure exploration
  - NIPS'15: online greedy learning
  - ICML'16: contextual combinatorial cascading bandits
  - NIPS'16: CMAB with general reward functions

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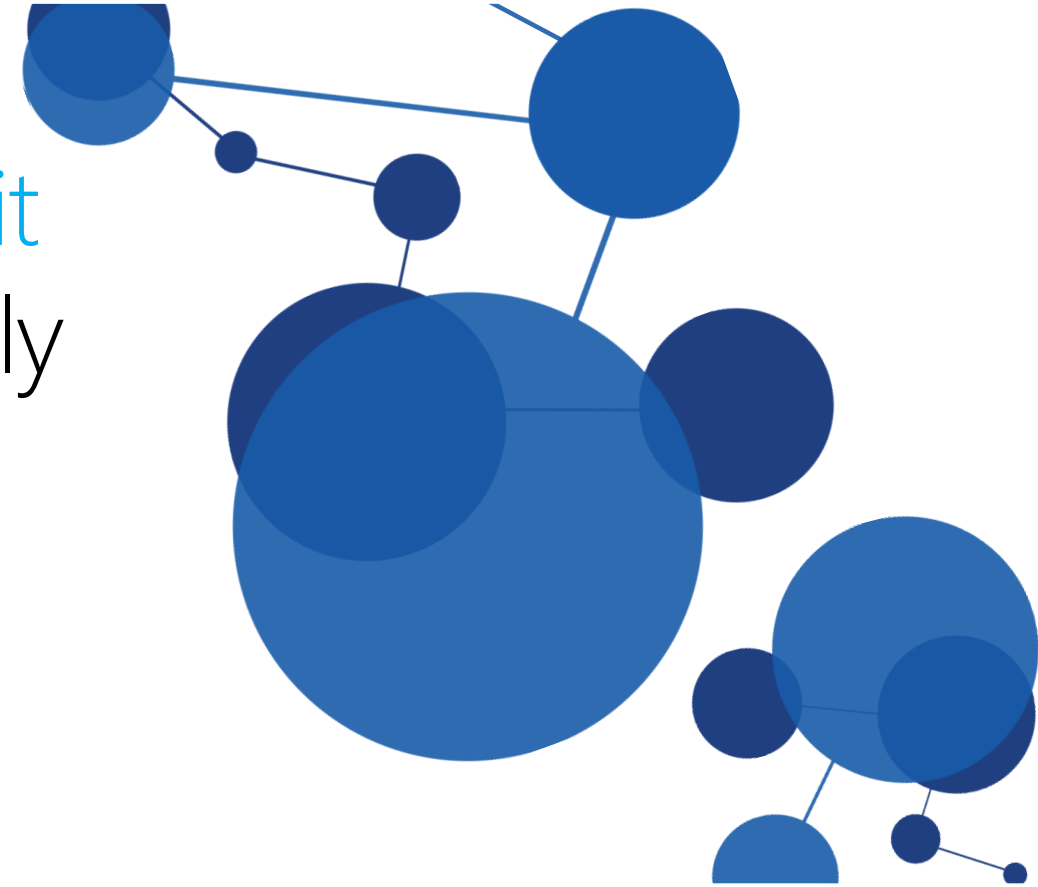
Combinatorial Multi-Armed Bandit and Its Extension to Probabilistically Triggered Arms

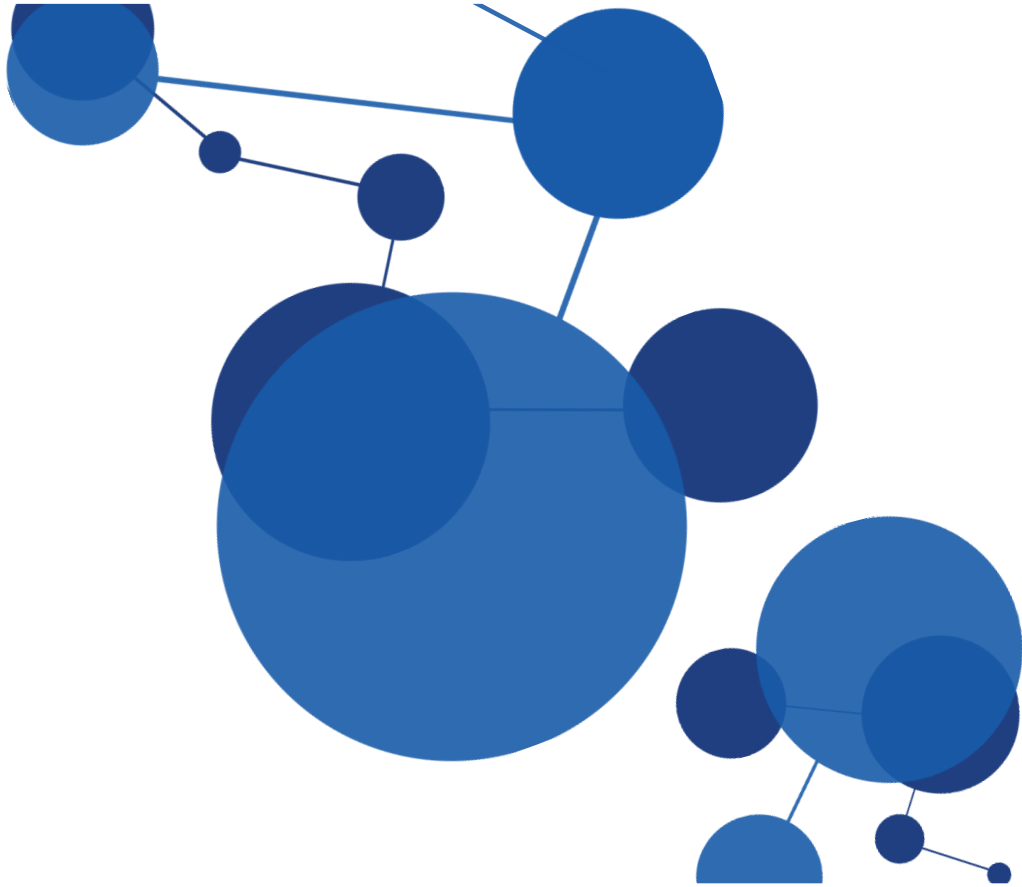


ICML'2013/JMLR'2016, joint work with
Yajun Wang, Microsoft
Yang Yuan, Cornell U.
Qinshi Wang, Tsinghua U.

Contribution of this work

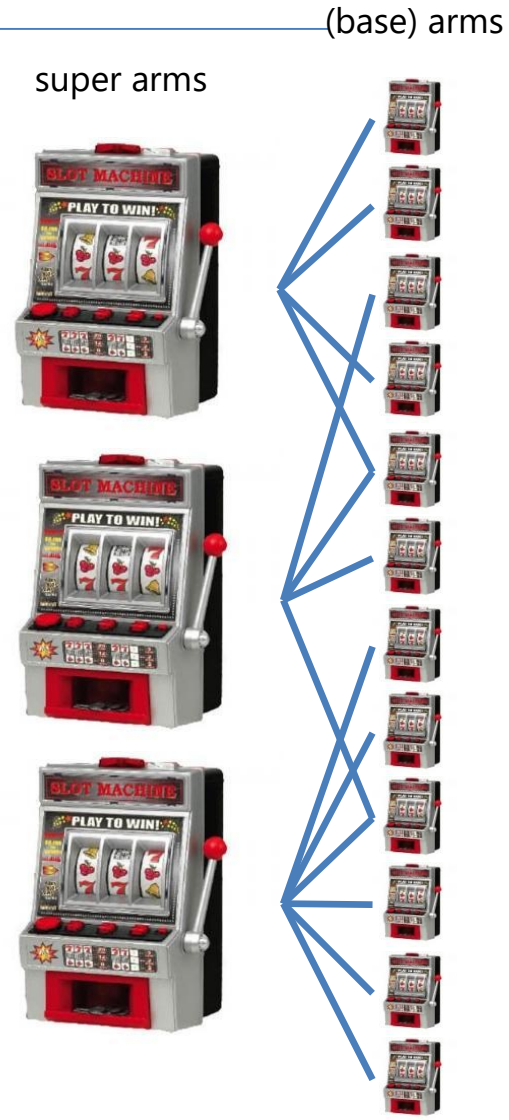
- Stochastic combinatorial multi-armed bandit framework
  - handling non-linear reward functions
  - UCB based algorithm and tight regret analysis
  - extend to probabilistically triggered arms
  - new applications using CMAB framework
- Comparing with related work
  - linear stochastic bandits [Gai et al. 2012]
    - CMAB is more general, and has much tighter regret analysis
  - online submodular optimizations (e.g. [Streeter& Golovin'08, Hazan&Kale'12])
    - for adversarial case, different approach
    - CMAB has no submodularity requirement

CMAB Framework



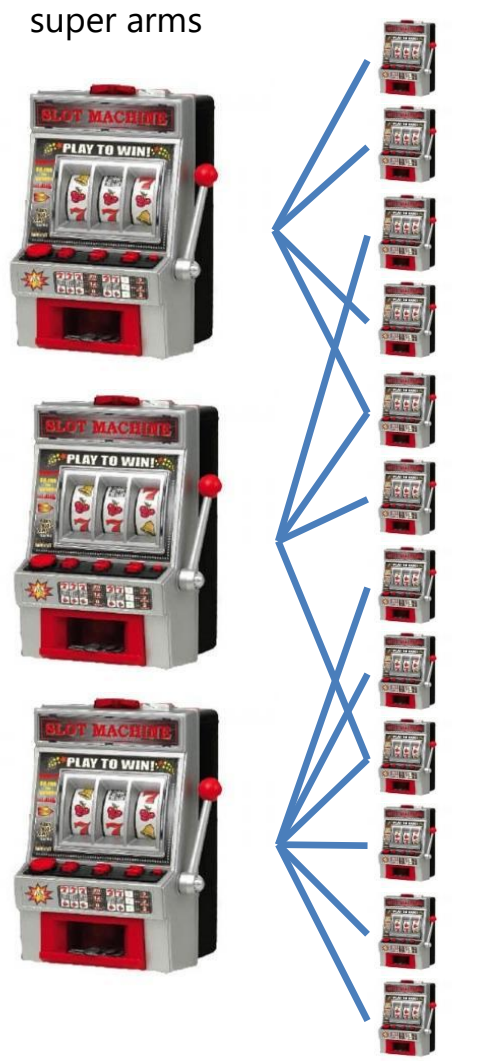
Combinatorial multi-armed bandit (CMAB) framework

- A super arm S is a set of (base) arms, $S \subseteq [m]$
- In round t, a super arm S_t^A is played according algo A
- When a super arm S is played, all based arms in S are played
- Outcomes of all played base arms are observed --- semi-bandit feedback
- Outcome of arm $i \in [m]$ has an unknown distribution with unknown mean μ_i



Rewards in CMAB

- Reward of super arm S_t^A played in round t, $R_t(S_t^A)$, is a function of the outcomes of all played arms
- Expected reward of playing arm S, $\mathbb{E}[R_t(S)]$, only depends on S and the vector of mean outcomes of arms, $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_m)$, denoted $r_{\boldsymbol{\mu}}(S)$
  - e.g. linear rewards, or independent Bernoulli random variables
  - generalization to be discussed later
- Optimal reward: $\mathbf{opt}_{\boldsymbol{\mu}} = \max_S r_{\boldsymbol{\mu}}(S)$



Handling non-linear reward functions --- two mild assumption on $r_{\boldsymbol{\mu}}(S)$

- Monotonicity
  - if $\boldsymbol{\mu} \leq \boldsymbol{\mu}'$ (pairwise), $r_{\boldsymbol{\mu}}(S) \leq r_{\boldsymbol{\mu}'}(S)$, for all super arm S
- Bounded smoothness
  - there exists a strictly increasing function $f(\cdot)$, such that for any two expectation vectors $\boldsymbol{\mu}$ and $\boldsymbol{\mu}'$,
    $|r_{\boldsymbol{\mu}}(S) - r_{\boldsymbol{\mu}'}(S)| \leq f(\Delta)$, where $\Delta = \max_{i \in S} |\mu_i - \mu_i'|$
  - Small change in $\boldsymbol{\mu}$ lead to small changes in $r_{\boldsymbol{\mu}}(S)$
    - A general version of Lipschitz continuity condition
- Rewards may not be linear, a large class of functions satisfy these assumptions

Offline computation oracle --- allow approximations and failure probabilities

- (α, β)-approximation oracle:
  - Input: vector of mean outcomes of all arms $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$,
  - Output: a super arm S, such that with probability at least β the expected reward of S under $\boldsymbol{\mu}$, $r_{\mu}(S)$, is at least α fraction of the optimal reward:

$$\Pr\left[r_{\boldsymbol{\mu}}(S) \geq \alpha \cdot \mathrm{opt}_{\boldsymbol{\mu}}\right] \geq \beta$$

(α, β)-Approximation regret

- Compare against the $\alpha\beta$ fraction of the optimal

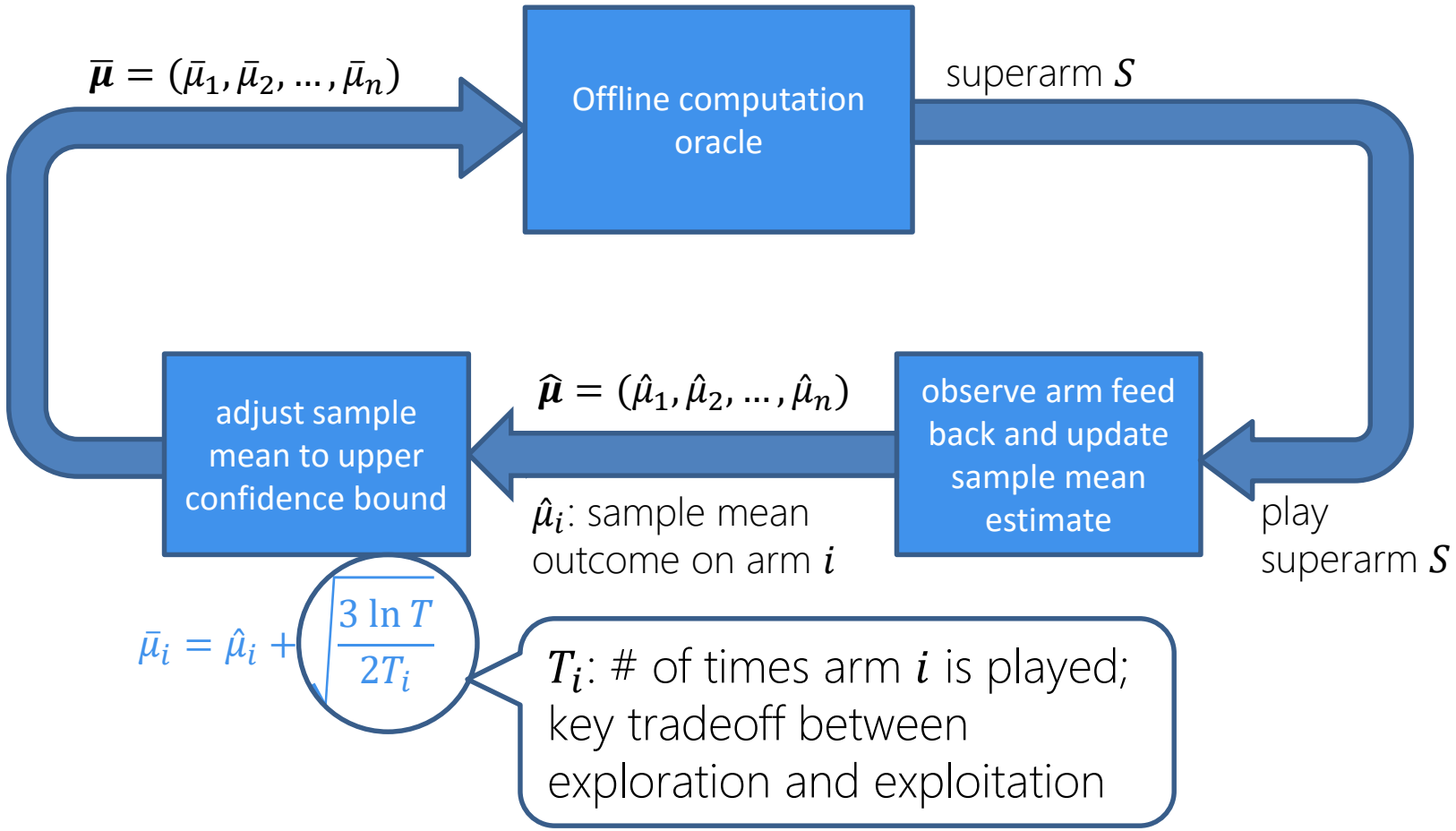
$$\text{Regret} = T \cdot \alpha\beta \cdot \mathbf{opt}_{\boldsymbol{\mu}} - \mathbb{E}[\sum_{i=1}^{T} r_{\boldsymbol{\mu}}(S_t^A)]$$

- Difficulty:
  - do not know arm outcome distribution
  - Oracle treatment: ignore
    - combinatorial structure
    - reward function
    - how oracle computes the solution

Classical MAB as a special case

- Each super arm is a singleton
- Oracle is taking the max, $\alpha = \beta = 1$
- Bounded smoothness function $f(x) = x$

Our solution: CUCB algorithm



Theorem 1: Gap-dependent bound

- The (α, β)-approximation regret of the CUCB algorithm in n rounds using an (α, β)-approximation oracle is at most

$$\sum_{i\in[n],\Delta_{\min}^{i}>0}\left(\frac{6\ln T\cdot\Delta_{\min}^{i}}{(f^{-1}(\Delta_{\min}^{i}))^{2}}+\int_{\Delta_{\min}^{i}}^{\Delta_{\max}^{i}}\frac{6\ln T}{(f^{-1}(x))^{2}}\mathrm{d}x\right)+\left(\frac{\pi^{2}}{3}+1\right)\cdot n\cdot\Delta_{\max}$$

  - $\Delta_{\min}^{i}$ ($\Delta_{\max}^{i}$) are defined as the minimum (maximum) gap between $\alpha\cdot\mathrm{opt}_{\mu}$ and reward of a bad super arm containing i.
    - $\Delta_{\min}=\min_{i}\Delta_{\min}^{i}$, $\Delta_{\max}=\max_{i}\Delta_{\max}^{i}$
    - Here, we define the set of bad super arms as $\mathcal{S}_{\mathrm{B}}=\{S\mid r_{\mu}(S)<\alpha\cdot\mathrm{opt}_{\mu}\}$
- When $f(x)=\gamma\cdot x$, regret bound: $O\left(\sum_{i}\frac{\gamma^{2}\ln T}{\Delta_{\min}^{i}}\right)$
- Match UCB regret for classic MAB

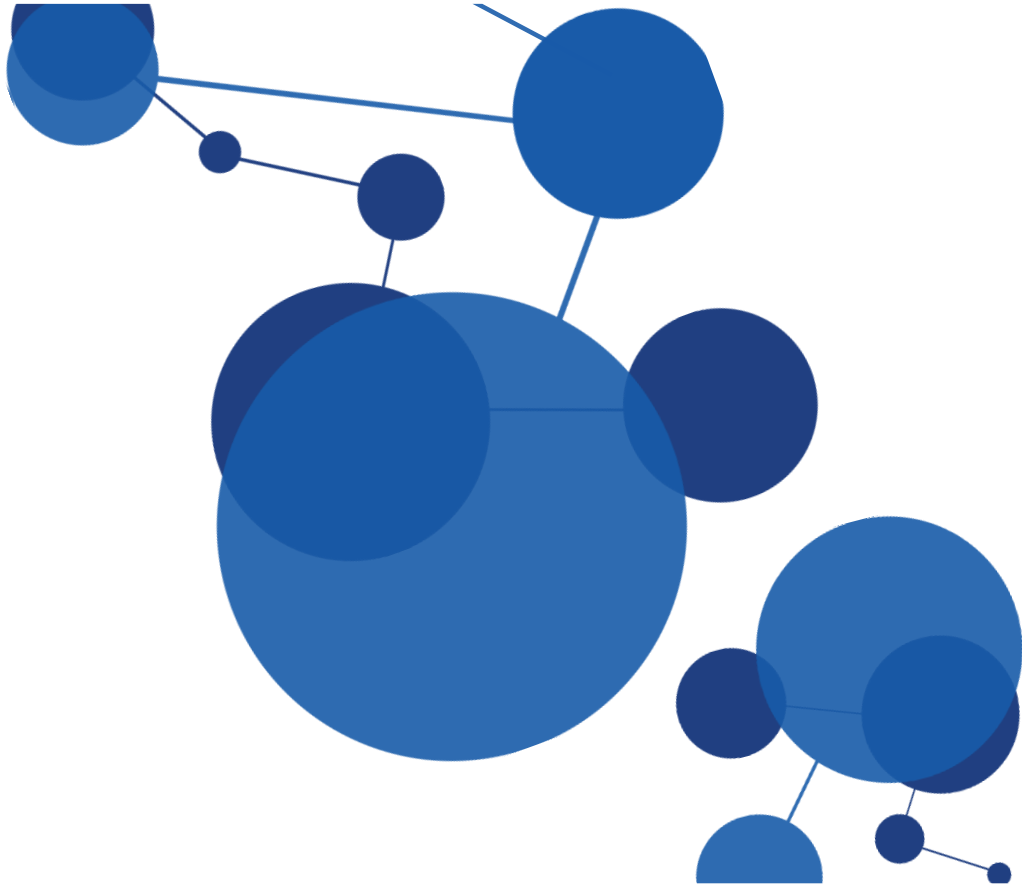
Theorem 2: Gap-free bound

- Consider a CMAB problem with an (α, β)-approximation oracle. If the bounded smoothness function $f(x) = \gamma \cdot x^{\omega}$ for some $\gamma > 0$ and $\omega \in (0,1]$, the regret of CUCB is at most:

$$\frac{2\gamma}{2-\omega} \cdot (6n \ln T)^{\frac{\omega}{2}} \cdot T^{1-\frac{\omega}{2}} + \left(\frac{\pi^2}{3} + 1\right) \cdot n \cdot \Delta_{\max}$$

- When $\omega = 1$, the gap-free bound is $O(\gamma\sqrt{nT \ln T})$

Applications of CMAB



Application to ad placement

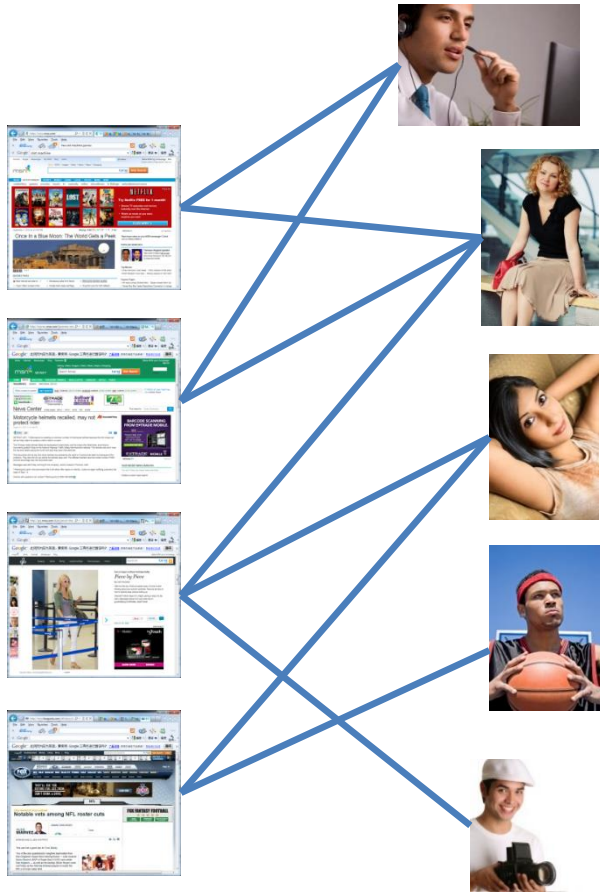
- Bipartite graph $G = (L, R, E)$
- Each edge is a base arm
- Each set of edges linking k webpages is a super arm
- Bounded smoothness function

$$f(\Delta) = |E| \cdot \Delta$$

- $(1 - {}^{1}/_{e}, 1)$-approximation regret

$$\sum_{i \in E, \Delta_{\min}^{i} > 0} \frac{12|E|^2 \ln T}{\Delta_{\min}^{i}} + \left(\frac{\pi^2}{3} + 1\right) \cdot |E| \cdot \Delta_{\max}$$

- improvement based on clustered arms is available



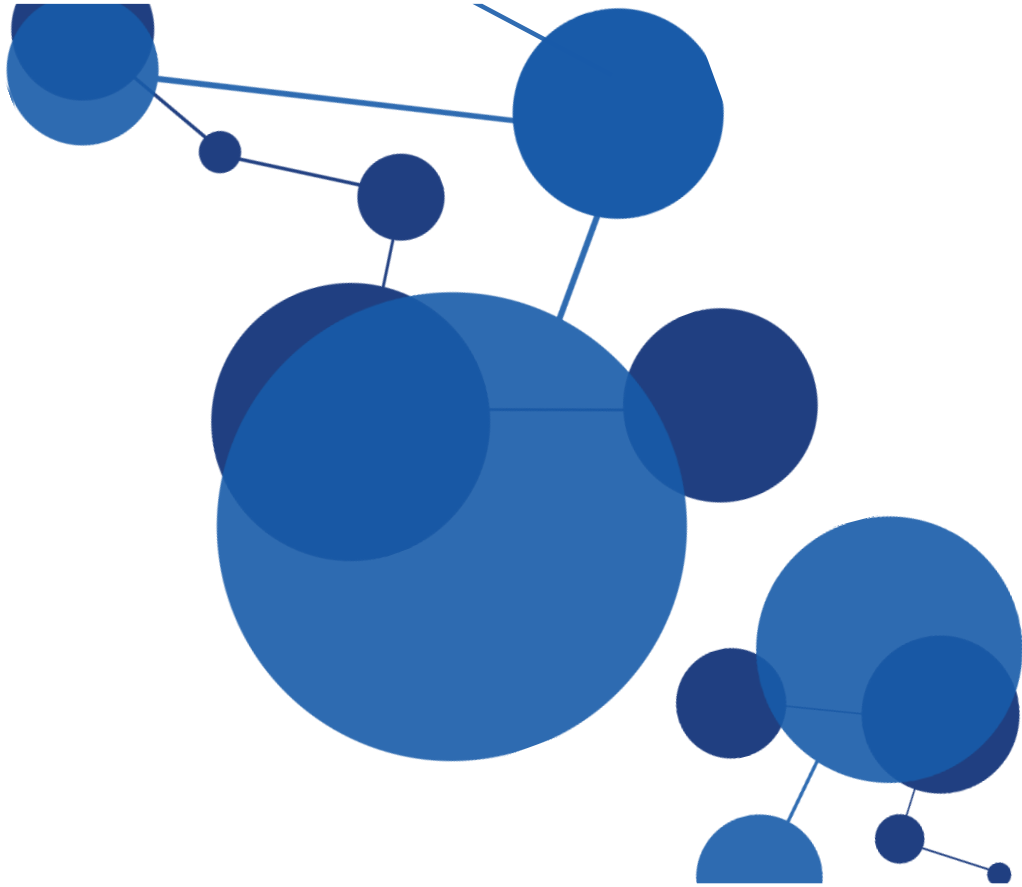
Application to linear bandit problems

- Linear bandits: matching, shortest path, spanning tree (in networking literature)
- Maximize weighted sum of rewards on all arms
- Our result significantly improves the previous regret bound on linear rewards [Gai et al. 2012]
  - Also provide gap-free bound

Application to social influence maximization

- Each edge is a base arm
- Require a new model extension to allow probabilistically triggered arms
  - Because a played base arm may trigger more base arms to be played --- the cascade effect
- Use the same CUCB algorithm
- Included in the journal version, JMLR'2016

Extensions and Variants



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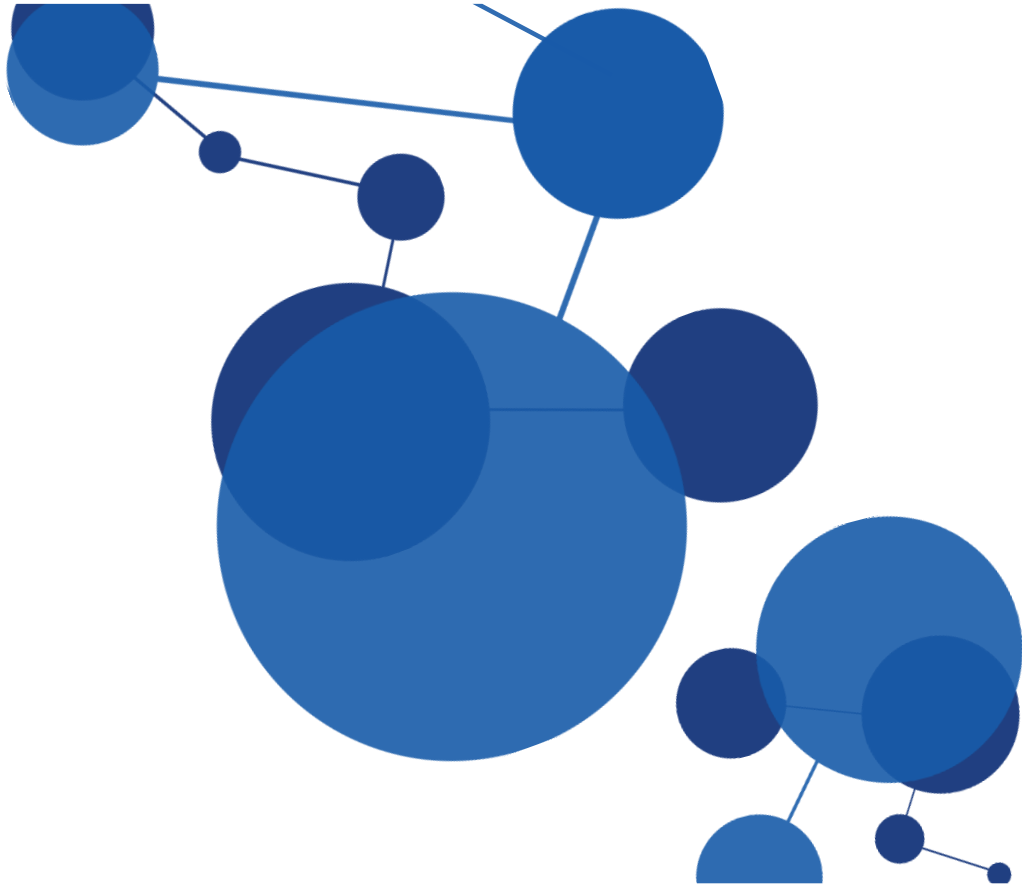
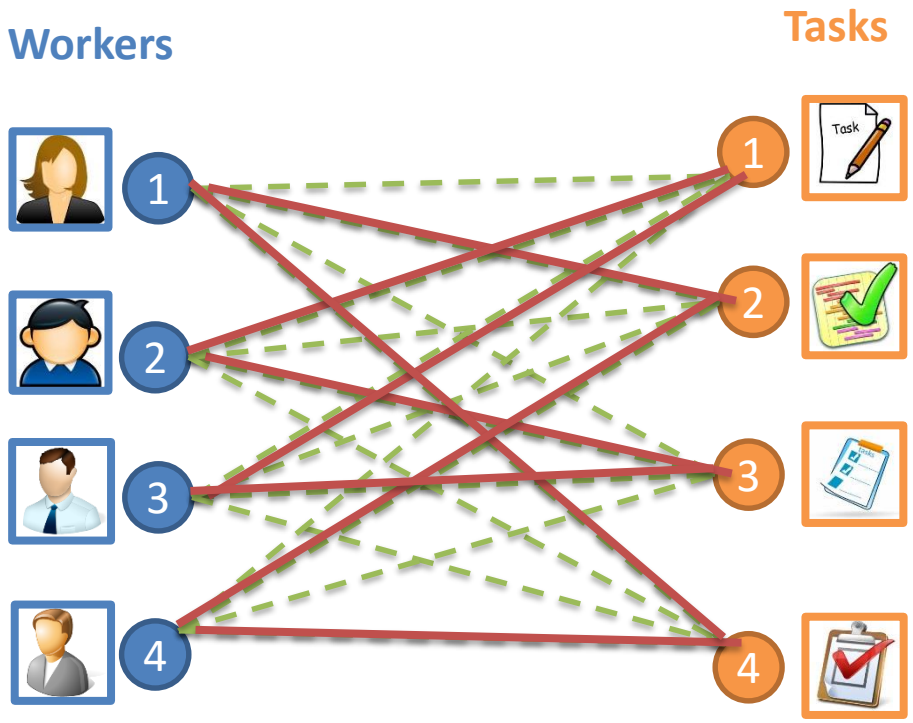
1. What if the feedback is limited?



Motivating example: Crowdsourcing

- Matching workers with tasks in a bipartite graph
  - Multiple timeslots: In each timeslot, assign one worker to one task, and the performance is probabilistic
  - Goal: cumulative reward from all timeslots and all worker-task pair performance
- Feedback may be limited:
  - workers may not report their performance
  - Some edges may not be observed in a round.
  - Feedback may or may not equal to reward



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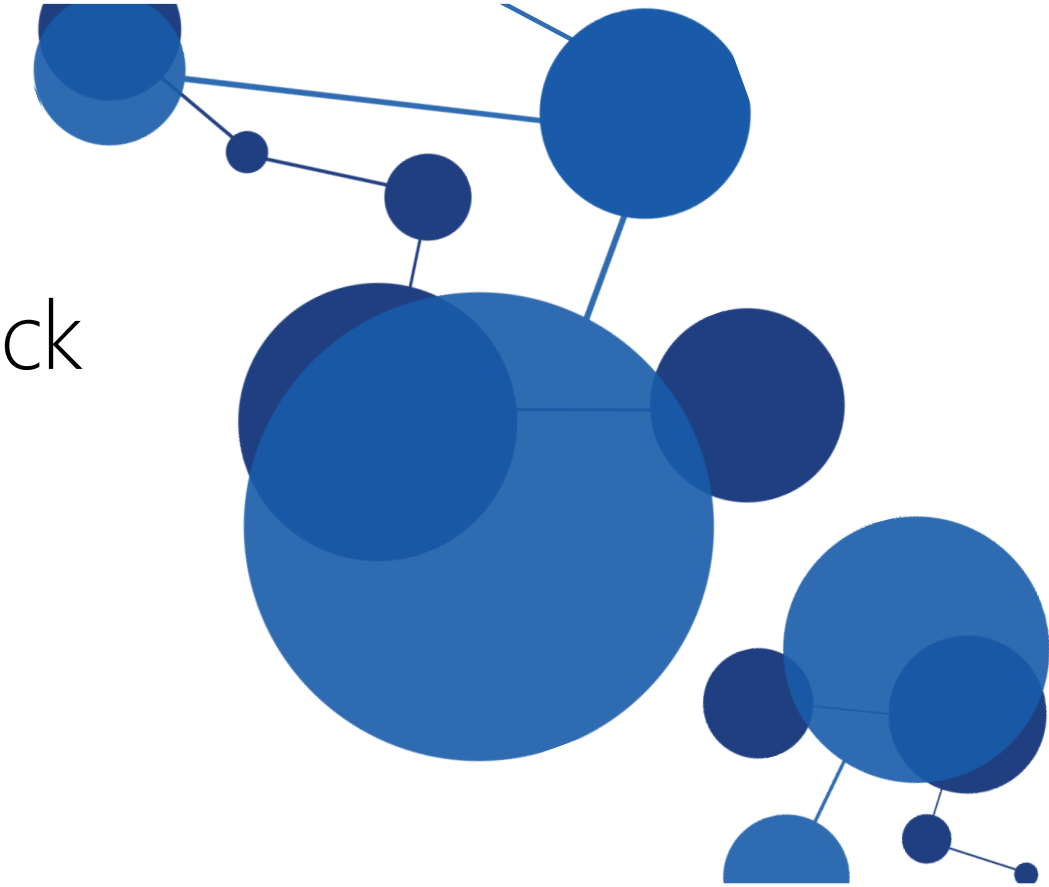

See ICML'14: Combinatorial Partial Monitoring Game with Linear Feedback and Its Applications

joint work with
Tian Lin, Tsinghua U.
Bruno Abrahao, Robert Kleinberg, Cornell U.
John C.S Lui, CUHK

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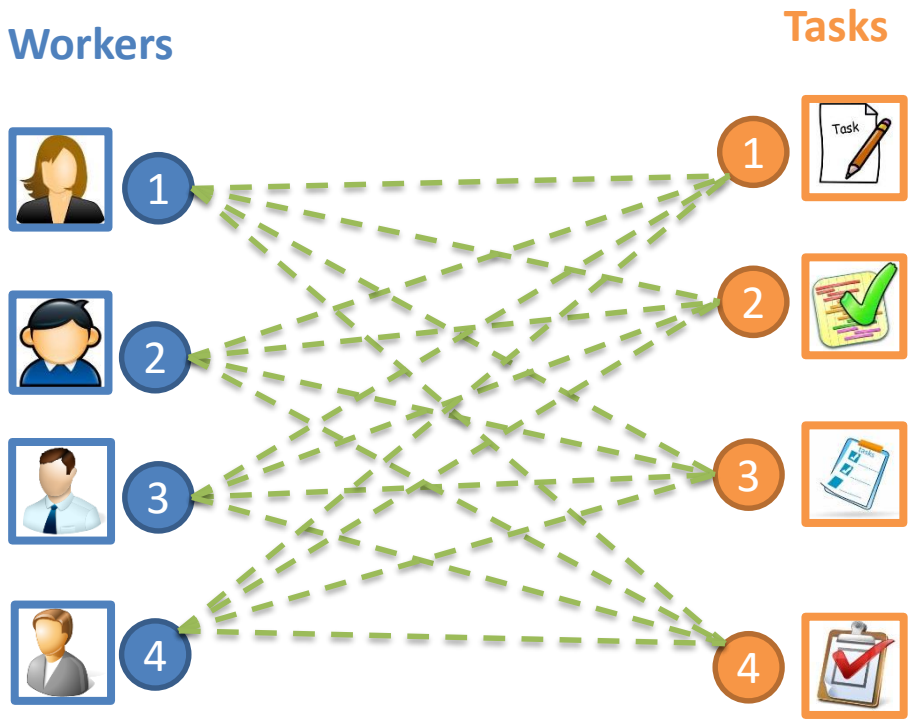
2. How to test base arms efficiently to find the best super arm?

Motivating example: Crowdsourcing

- Matching workers with tasks in a bipartite graph
  - Initial test period: adaptively test worker-task pair performance
  - Goal: at the end of test period, find the best worker-task matching



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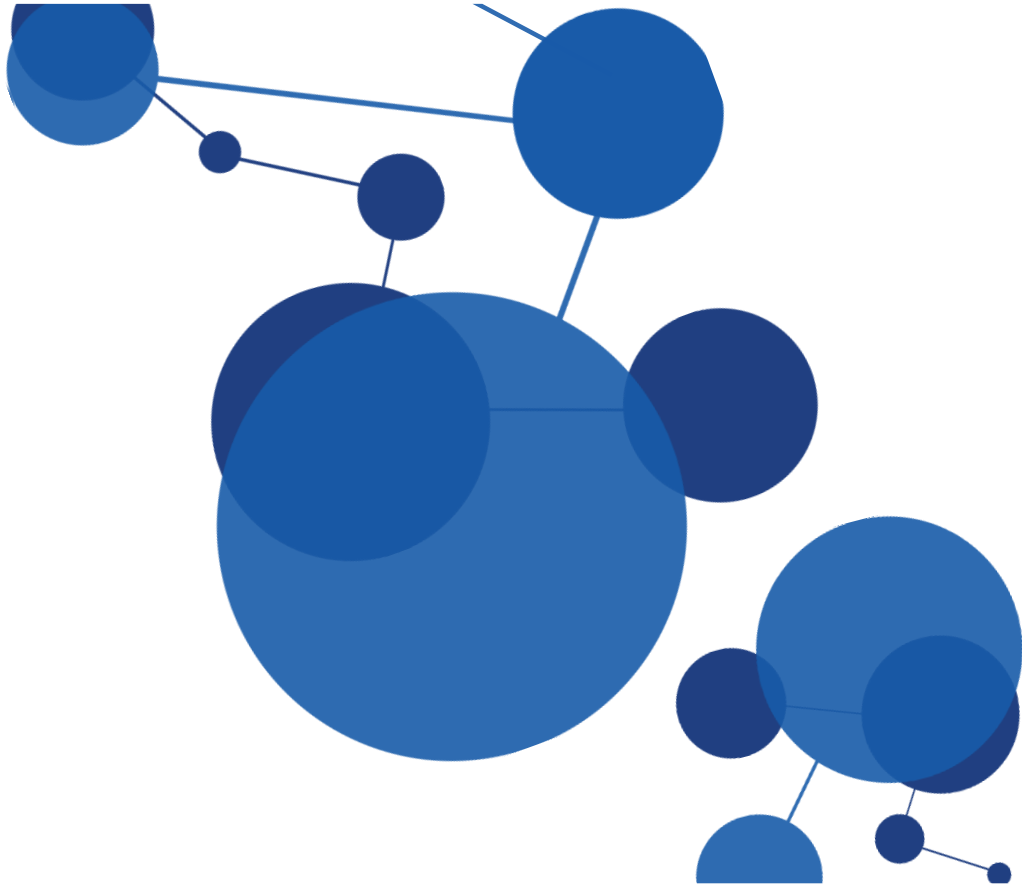
See NIPS'14: Combinatorial Pure Exploration in Multi-Armed Bandits



joint work with
Shouyuan Chen, Irwin King, Michael R. Lyu, CUHK
Tian Lin, Tsinghua U.

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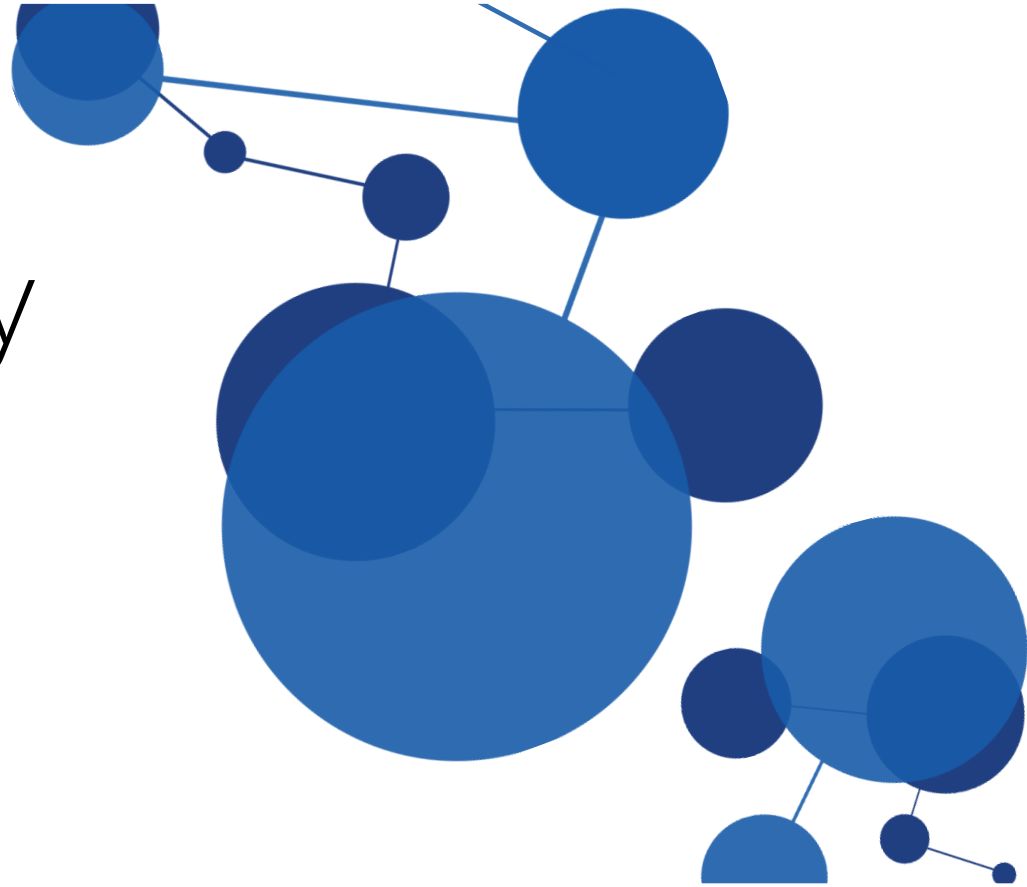
3. How to turn offline greedy algorithm to online greedy?



Motivation

- Greedy algorithm is extensively used in optimizations as approximation algorithms or heuristics
- Is there a systematic way of turn offline greedy algorithm into an online greedy learning algorithm?
- Can be viewed as open up the offline oracle to help learning

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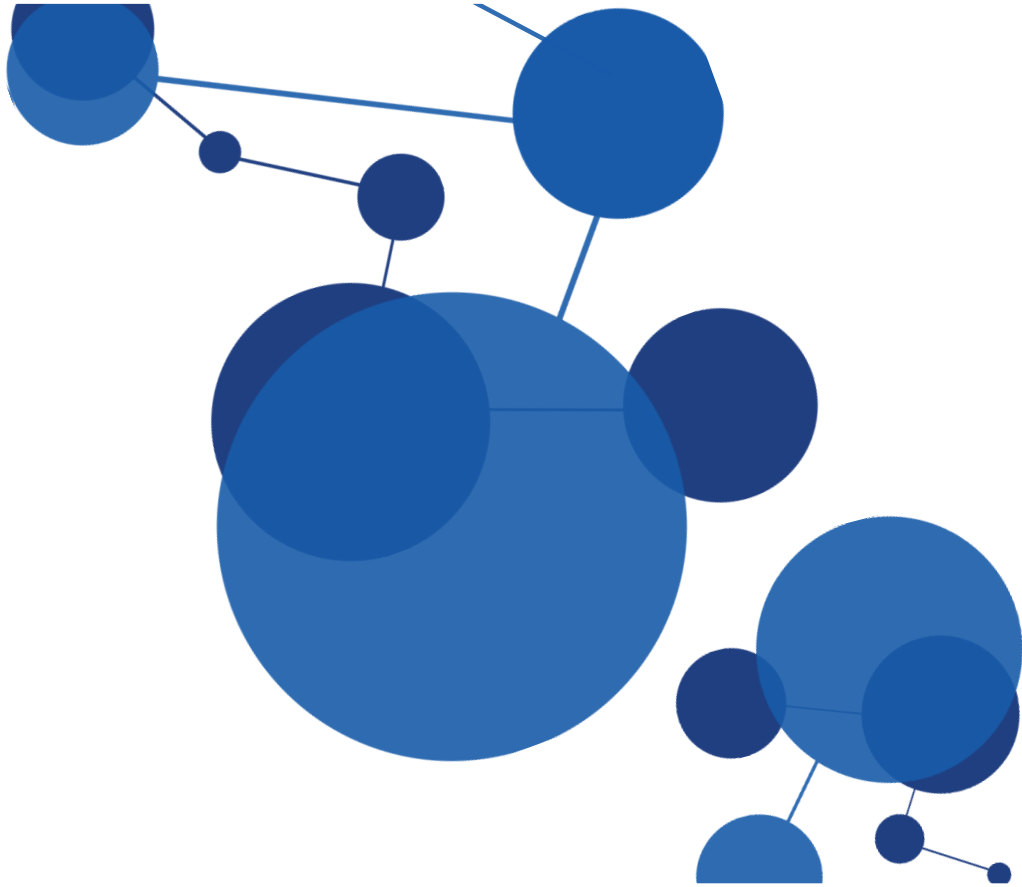
See NIPS'15: Stochastic Online Greedy Learning with Semi-bandit Feedbacks



joint work with
Tian Lin, Jian Li, Tsinghua U.

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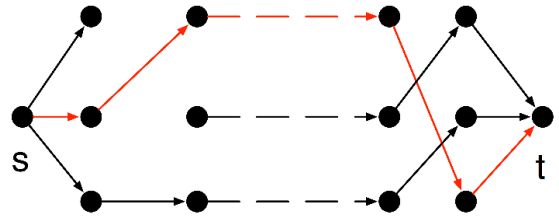
4. What if estimating means of arms is not enough?

Motivating example: graph routing

- Expected Utility Maximization (EUM) Model
  - Each edge i has a random delay X_i
  - Each routing path is a subset of edges, S
  - utility of a routing path S: $u(\sum_{i \in S} X_i)$
    - $u(\cdot)$ is nonlinear, modeling risk-averse or risk-prone behavior
  - Goal: maximize $\mathbb{E}[u(\sum_{i \in S} X_i)]$
- Issue for online learning (when distributions of X_i's are unknown
  - only estimating the mean of X_i is not enough



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See NIPS'16: Combinatorial Multi-Armed Bandit with General Reward Functions

joint work with
Wei Hu, Princeton U.
Fu Li, U. of Texas at Austin
Jian Li, Yu Liu, Tsinghua U
Pinyan Lu, Shanghua U. of Finance and Economics

Overall summary

- Central theme
  - deal with stochastic and unknown inputs for combinatorial optimization problems
  - modular approach: separate offline optimization with online learning
    - learning part does not need domain knowledge on optimization
- More wait to be done
  - Many other variants of combinatorial optimizations problems --- as long as it has unknown inputs need to be learned
  - E.g., nonlinear rewards, approximations, adversarial unknown inputs, etc.

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Thank you!