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PROBABILISTIC PROGRAMMING

Microsoft

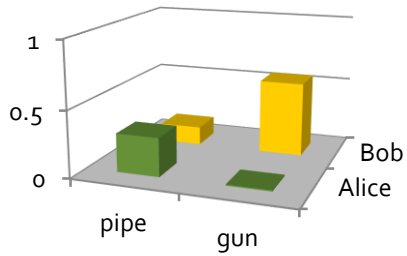
Machine Learning and Programming

- "Data widely available; what is scarce is the ability to extract wisdom from them" , Hal Varian, 2010
- "Machine learning!", Mundie and Schmidt at Davos, 2012

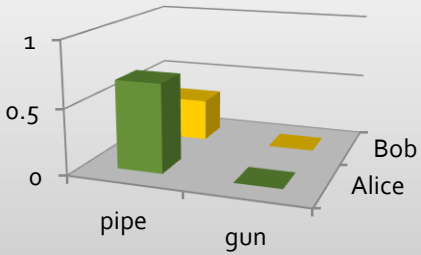
- Researchers use Bayesian statistics as unifying principle:
  - Models are conditional probabilities; inference algorithms separate
- For the programmer, what's the problem?
  - Cottage industry of inflexible libraries and algorithms
  - Custom implementations are 1000s LOC
- Probabilistic programming offers a solution
  - Write your model as succinct, adaptable probabilistic program
  - Run compiler to get efficient inference code

Murder Mystery in Fun

```
// Either Alice or Bob dunnit
// Alice dunnit 30%, Bob dunnit 70%
// Alice uses gun 3%, uses pipe 97%
// Bob uses gun 80%, uses pipe 20%
let mystery () =
  let aliceDunnit = random (Bernoulli 0.30)
  let withGun =
    if aliceDunnit
    then random (Bernoulli 0.03)
    else random (Bernoulli 0.80)
  aliceDunnit, withGun
```




```
// Pipe at scene - now Alice dunnit 69%
let PipeFoundAtScene () =
  let aliceDunnit, withGun = mystery ()
  observe(withGun = false)
  aliceDunnit, withGun
```



Probabilistic Programming

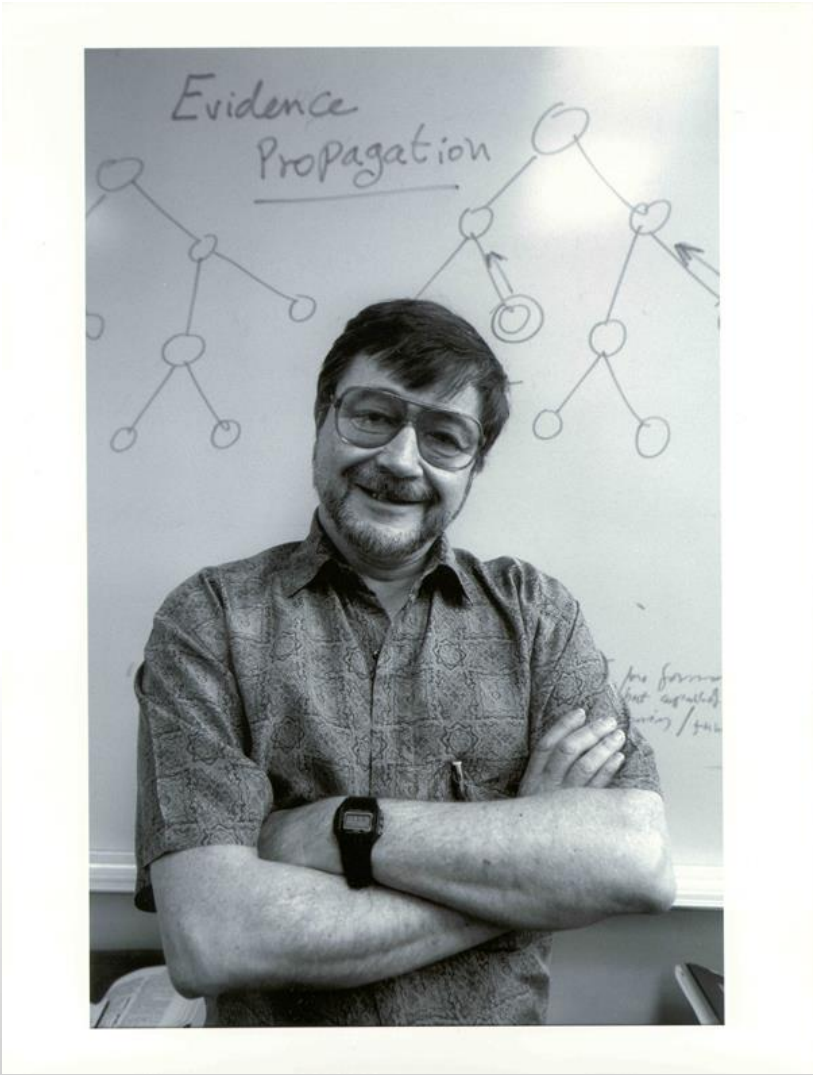
- BUGS (Spiegelhalter et al 1994, CU)
- IBAL (Pfeffer, 2002)
- BLOG (Milch et al 2005, UCB/MIT) – Gibbs sampling
- Alchemy (Domingos et al 2005, UW) – probabilistic logic programming
- CHURCH (Goodman et al 2008, MIT) – recursive probabilistic functional programming
- HANSEI (Kiselyov and Shan, 2009) – discrete distributions from Ocaml
- FACTORIE (McCallum et al 2008, UMASS)
- Infer.NET

Judea Pearl, Turing Award Winner 2011

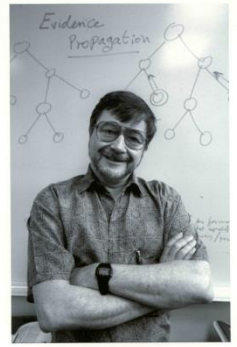
For fundamental contributions to artificial intelligence through the development of a calculus for probabilistic and causal reasoning.

…

He identified uncertainty as a core problem faced by intelligent systems and developed an algorithmic interpretation of probability theory as an effective foundation for the representation and acquisition of knowledge.



Probabilistic Graphical Models



- Pioneered by Bayes Networks (Pearl 1988)
  - **Model** of world, both observed and unobserved states
  - **Probabilistic** for uncertainty: missing data, noise, how data arises
  - **Graphical** notations capture dependence, for scalability
- Pearl "invented message-passing algorithms that exploit graphical structure to perform probabilistic reasoning effectively"
- Many application areas: "natural language processing, speech processing, computer vision, robotics, computational biology, and error-control coding"
- In last few years, large-scale deployments include:
  - TrueSkill – How do we rank Halo players?
  - AdPredictor – How likely is a user to click on this ad?

Infer.NET (since 2006)

- A .NET library for probabilistic inference
  - Multiple inference algorithms on graphs
  - Far fewer LOC than coding inference directly
  - Designed for large scale inference
  - User extensible
- Supports rapid prototyping and deployment of Bayesian learning algorithms
  - Graphs represented by object model for pseudo code, but not as runnable code
- **Realization**: language geeks can do machine learning, without comprehensive understanding of Bayesian stats, message-passing, etc


infer.net
Microsoft
Visual F#

Infer.NET Fun – New Feature

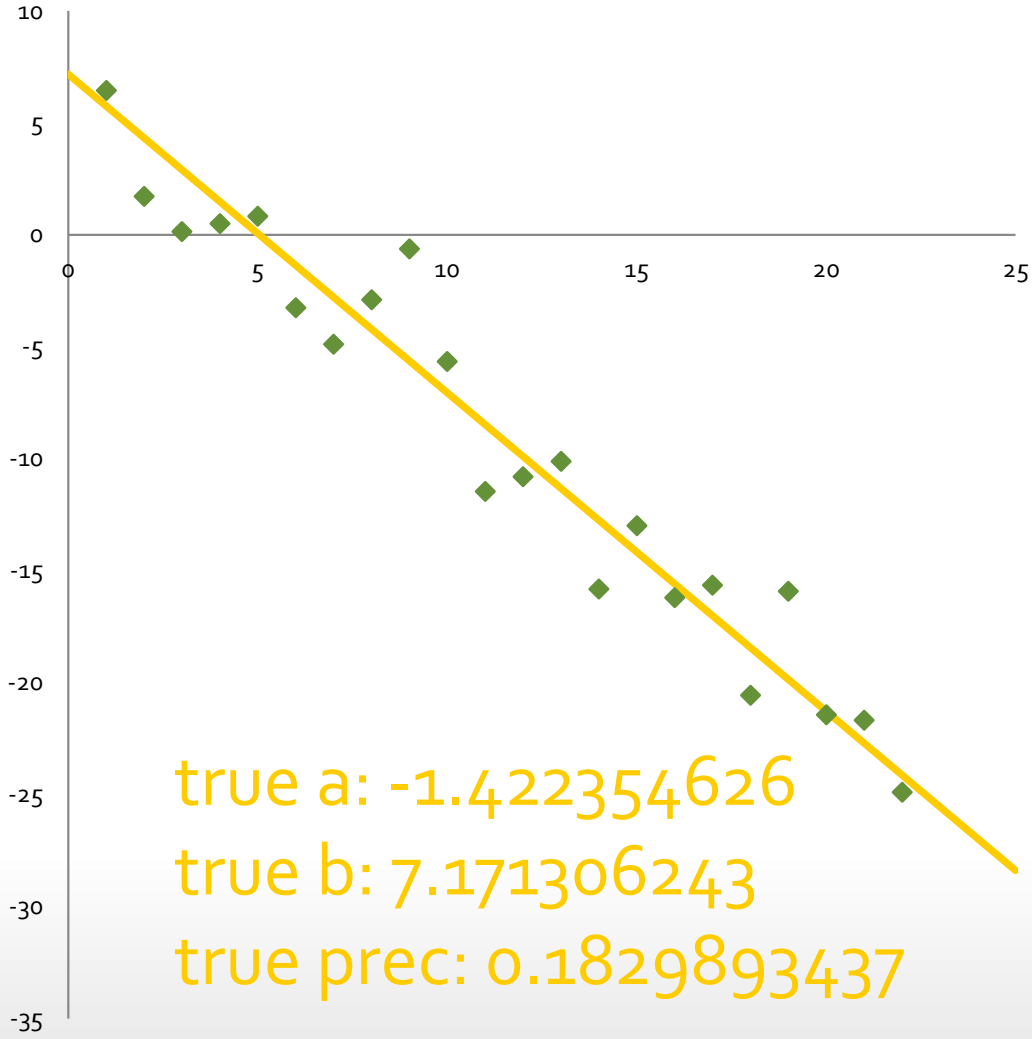

- Bayesian inference by functional programming
  - Write your model in F#
  - Run forwards to synthesize data
  - Run backwards to infer parameters

- Benefits:
  - Models are simply code in F#'s simple succinct syntax
  - Higher-level features than C# OM: tuples, records, array comprehensions,functions
  - Custom graphical notations ("plates","gates") just code
  - Testing inference by running forwards then backwards

http://research.microsoft.com/fun

Programming in Infer.NET Fun

Linear Regression



true a: -1.422354626

true b: 7.171306243

true prec: 0.1829893437

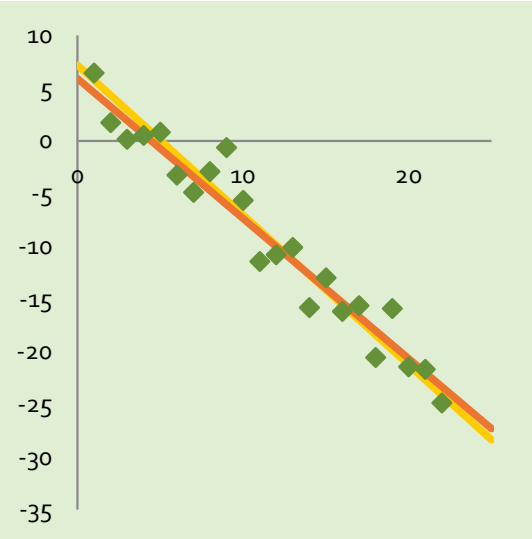
- Linear regression:

  Forwards, compute $y_i = ax_i + b + noise$ from a and b

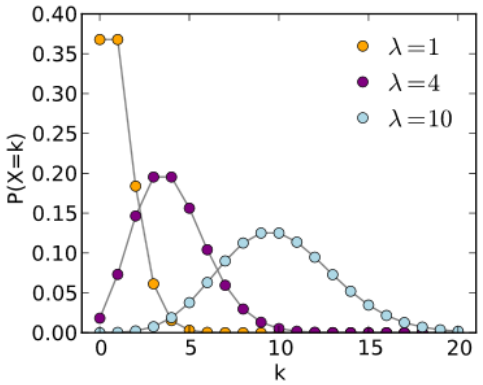
  Backwards, given y_i infer a and b

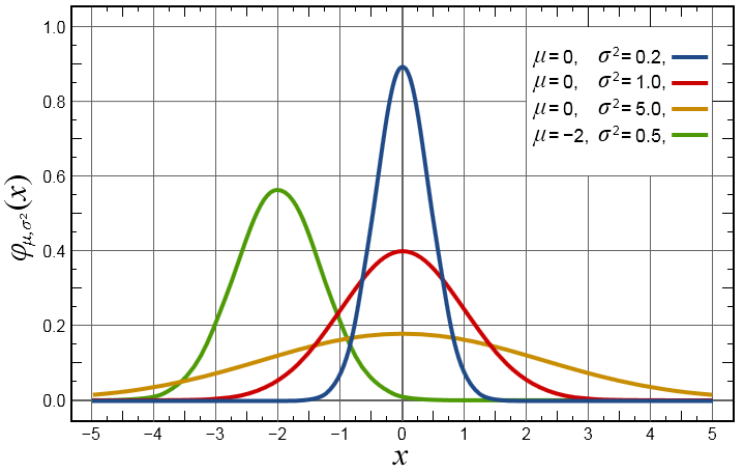
Linear Regression in Fun

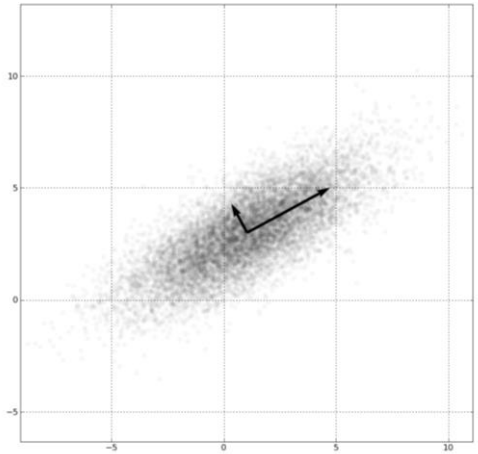
```
let prior() =
  let a = random(Gaussian(0.0, 1.0))
  let b = random(Gaussian(5.0, 0.3))
  let noise = random (Gamma(1.0, 1.0))
  a, b, noise

let point x a b noise =
  x, random(Gaussian(a * x + b, noise))

let model data =
  let a, b, noise = prior()
  observe (data =
    [| for x,_ in data -> point x a b noise |])
  a, b, noise

let aD, bD, noiseD = inferFun3 <@ model @> data
```

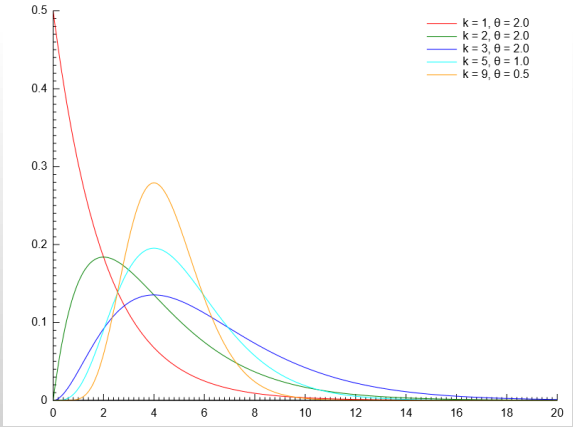


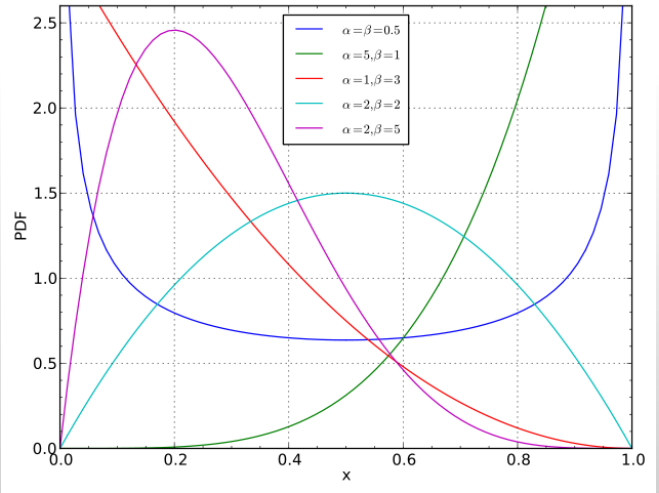
Some Probability Distributions in Fun

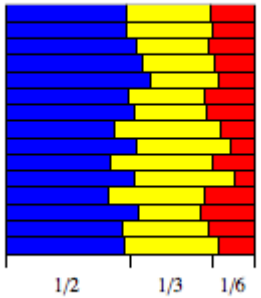












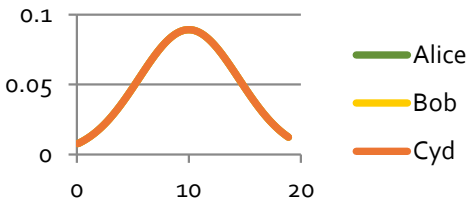
Source: Wikipedia

```
type ::=    // Fun value type
    unit
    bool
    int
    double
    (type1 * ... * typeN)
    { field1: type1; ...; fieldN: typeN}
    type[]

dist ::=    // Fun distribution
    Beta(expr)
    Gaussian(expr1,expr2)
    Gamma(expr1,expr2)
    Binomial(expr1,expr2)
    VectorGaussian(expr1,expr2)
    Discrete(expr)
    Poisson(expr)
    Bernoulli(expr)
    Dirichlet(expr)
    Wishart(expr1,expr2)
```

```
expr ::=    // Fun expression
  var                                  // variable
  literal                              // literal eg -1.0, true, 42
  { field1 = expr1; ...; fieldN = exprN }  // record
  ( expr1, ..., exprN )                // tuple
  expr.field                           // record lookup
  fst(expr)                            // first projection
  snd(expr)                            // second projection
  not expr                             // negation
  expr1 R expr2                        //  relation (eg, =, >)
  expr1 f expr2                        // function (eg, +, -)
  let var = expr1 in expr2             // let
  if expr1 then expr2 else expr3       // conditional
  expr : type                          // type annotation
  for var in expr1 do expr2            // iteration loop
  [| 0 .. expr |]                      // integer range
  [| for var in expr1 -> expr2 |]      // comprehension
  Array.zip expr1 expr2                // zip two arrays
  random(dist)                         // draw from distribution
  observe expr                         // observation of boolean
```

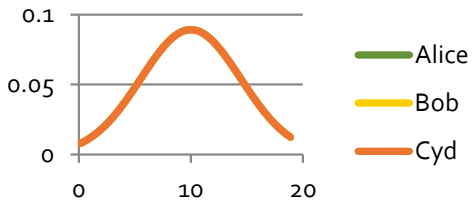
TrueSkill in Fun

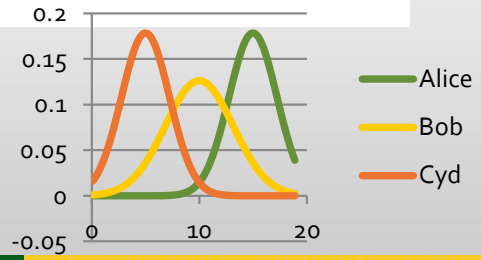
```
// prior distributions, the hypothesis
let skill() = sample (Gaussian(10.0,20.0))
let Alice,Bob,Cyd = skill(),skill(),skill()
```



TrueSkill in Fun

```
// prior distributions, the hypothesis
let skill() = sample (Gaussian(10.0,20.0))
let Alice,Bob,Cyd = skill(),skill(),skill()
// observe the evidence
let performance player = sample (Gaussian(player,1.0))
observe (performance Alice > performance Bob) //Alice beats Bob
observe (performance Bob > performance Cyd) //Bob beats Cyd
observe (performance Alice > performance Cyd) //Alice beats Cyd
// return the skills
Alice,Bob,Cyd
```

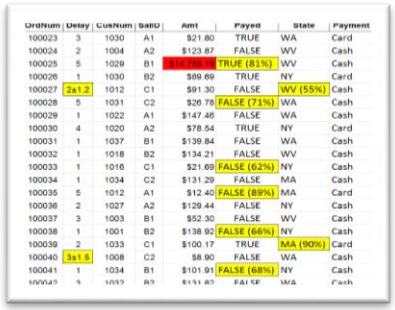




The **model-learner** pattern brings structure and types, as well as PL syntax, to probabilistic graphical models

```
module LinearRegression =
  type TH = {MeanA: double; PrecA: double; … }
  let h = {MeanA=0.0; PrecA=1.0; … }
  type TW<'a,'b,'c> = {A:'a; B:'b; Noise:'c}
  type TX = double
  type TY = double
  let M: Model<TH,TW<double,double,double>,TX,TY> =
    { Prior = <@ fun h ->
        { A = random(Gaussian(h.MeanA,h.PrecA))
          B = random(Gaussian(h.MeanB,h.PrecB))
          Noise = random(Gamma(h.ShapeN,h.ScaleN)) } @>
      Gen = <@ fun a -> let m = (a.W.A * a.X) + a.W.B
                  random(Gaussian(m, a.W.Noise)) @> }
```

```
module Classifier
module Regression
module TrueSkill
module TopicModel
```



Write your model in F# or C#

Or choose from library

Or automatically generate

Assemble multiple models

```
type Model
```

Choose algorithm (eg, EP, VMP, Gibbs, ADD, Filzbach)

Synthetic data to test learner

```
type ISampler
```

```
type ILearner
```

Train, predict, repeat

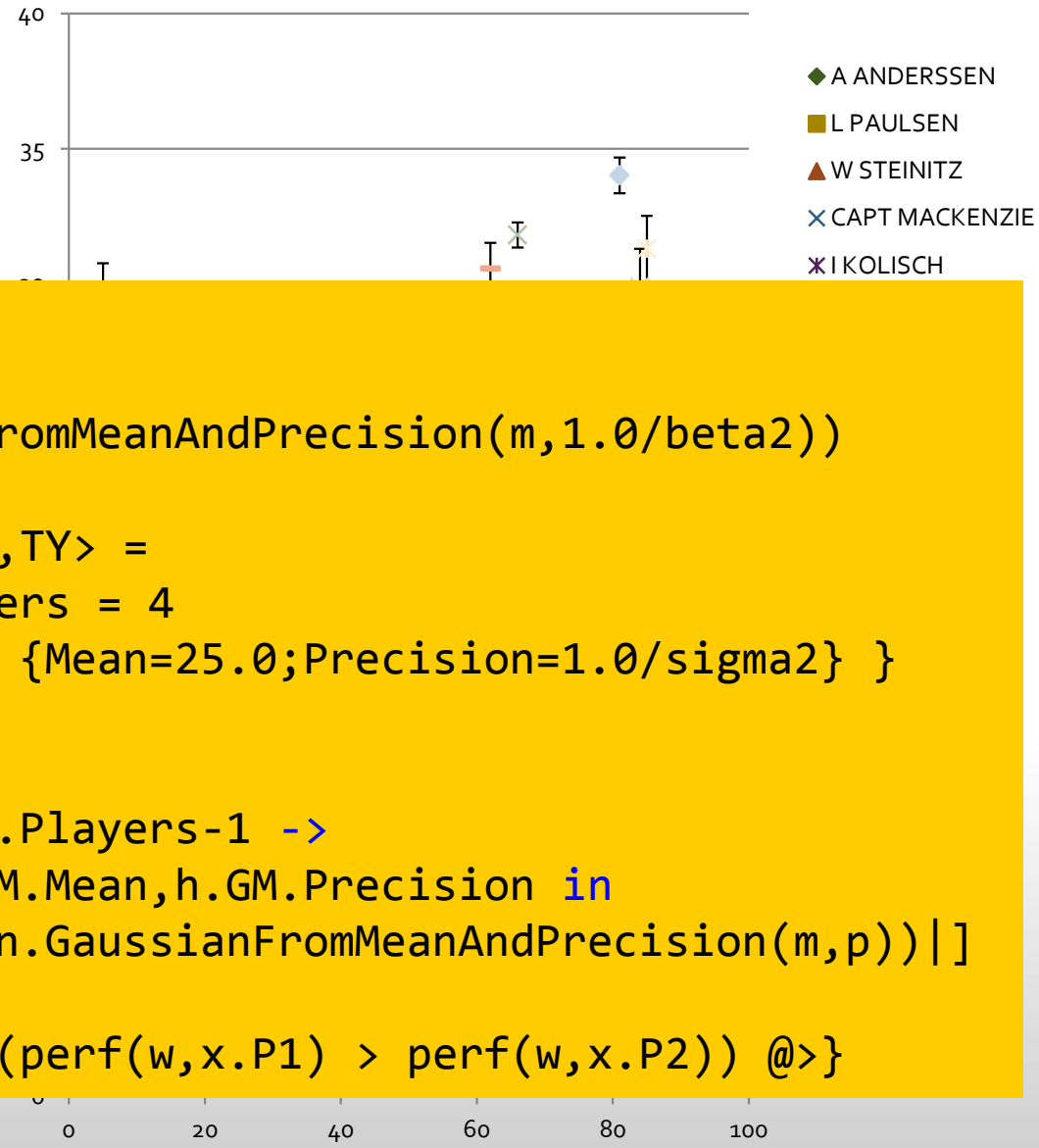
Models, Samplers, and Learners

```
type Model<'TH,'TW,'TX,'TY> =
  { HyperParameter: 'TH
    Prior: Expr<'TH ->'TW>
    Gen: Expr<'TW *'TX ->'TY> }
```

```
type ISampler<'TW,'TX,'TY> =
  interface
    abstract Parameters: 'TW
    abstract Sample: x:'TX -> 'TY
  end
```

```
type ILearner<'TDistW,'TX,'TY,'TDistY> =
  interface
    abstract Train: x:'TX * y:'TY -> unit
    abstract Posterior: unit -> 'TDistW
    abstract Predict: x:'TX -> 'TDistY
  end
```

TrueSkill



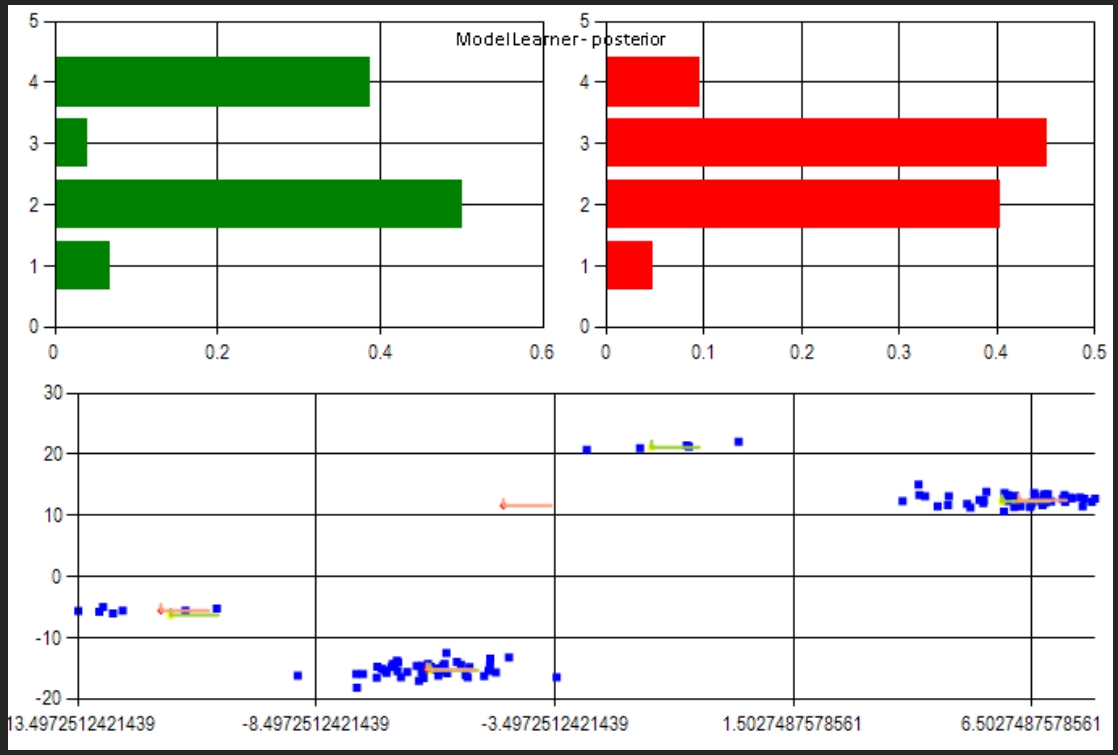

```
let perf(w,pid) =
  let m = w.Skills.[pid]
  Fun.random(Fun.GaussianFromMeanAndPrecision(m,1.0/beta2))

let M:Model<TH,TW<real>,TX,TY> =
  { HyperParameter = {Players = 4
                      GM = {Mean=25.0;Precision=1.0/sigma2} }
    Prior = <@ fun h ->
      {Skills =
          [| for x in 0..h.Players-1 ->
             let m,p = h.GM.Mean,h.GM.Precision in
             Fun.random(Fun.GaussianFromMeanAndPrecision(m,p))|]
      } @>
    Gen = <@ fun (w,x) -> (perf(w,x.P1) > perf(w,x.P2)) @>}
```

Binary Mixture Combinator

- We code a variety of idioms as functions from models to models, eg, mixtures:

```
let Mixture(m1,m2) =
    {Prior =
      <@ fun h ->
         {Bias=random(Uniform(0.0,1.0))
          P1=(%m1.Prior) h
          P2=(%m2.Prior) h} @>
     Gen =
      <@ fun (w,x) ->
          if random(Bernoulli(w.Bias))
          then (%m1.Gen) (w.P1,x)
          else (%m2.Gen) (w.P2,x) @>}
```

Mixture Of Gaussians



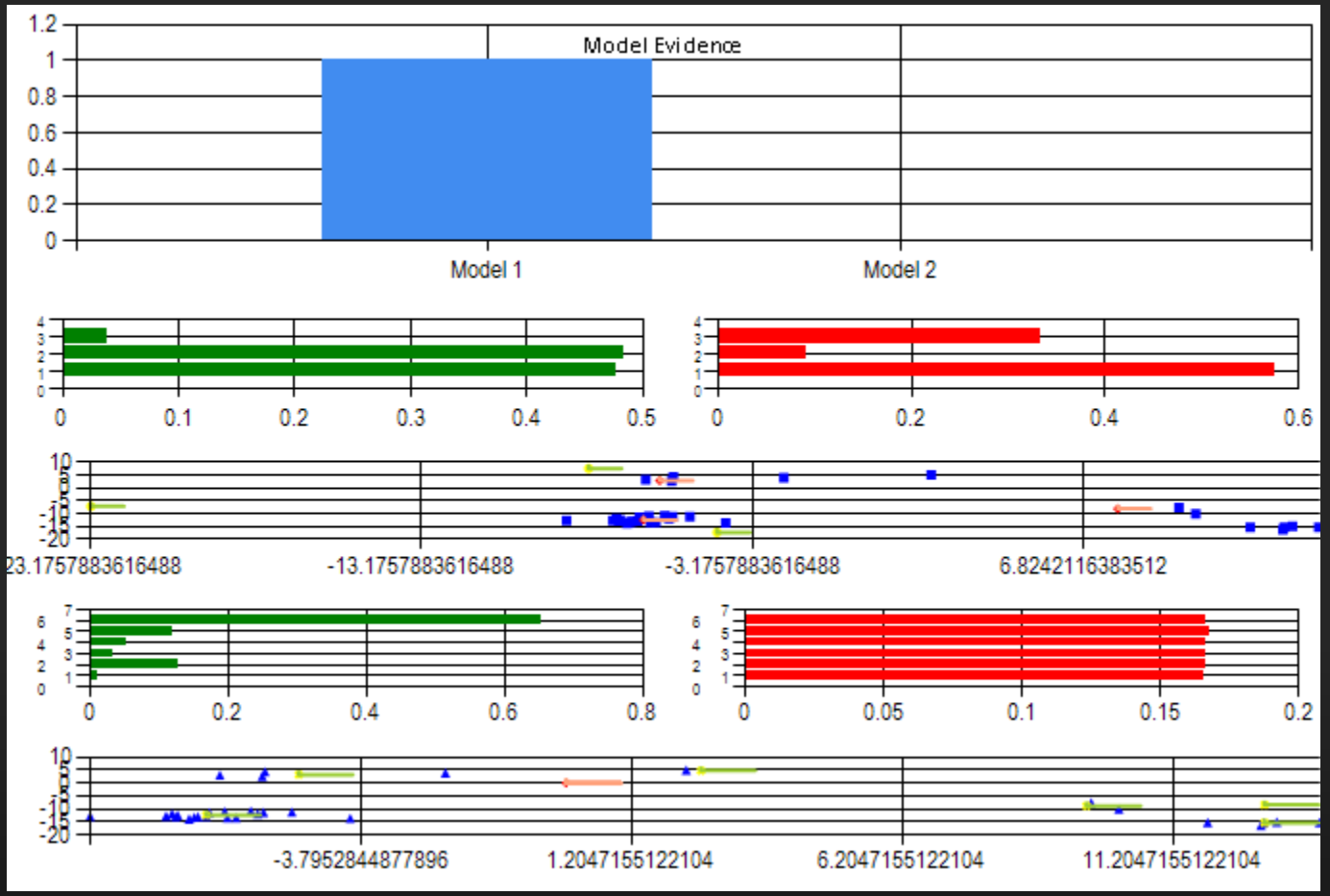

```
let k = 4 // number of clusters in the model
let M = IIDArray.M(KwayMixture.M(VectorGaussian.M,k))

let sampler1 = Sampler.FromModel(M);
let xs = [| for i in 1..100 -> () |]
let ys = sampler1.Sample(xs);

let learner1 = InferNetLearner.LearnerFromModel(M,mg0)
do  learner1.Train(xs,ys)
let (meansD2,precsD2,weightsD2) = learner1.Posterior()
```

Evidence Combinator

- A variation of mixtures, where the choice between models is made per-model, rather than per-output

```
let Evidence(m1,m2) =
  {Prior = <@ fun (bias,h1,h2) ->
                (random(Bernoulli(bias))),
                 (%m1.Prior) h1, (%m2.Prior) h2) @>
   Gen = <@ fun ((switch,w1,w2),x) ->
              if switch then (%m1.Gen) (w1,x)
              else (%m2.Gen) (w2,x) @>}
```

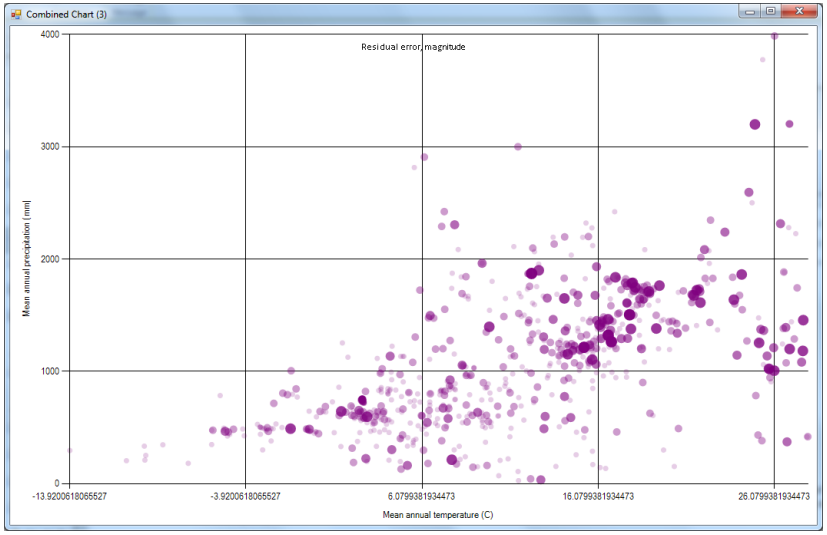
Demo: Model Selection

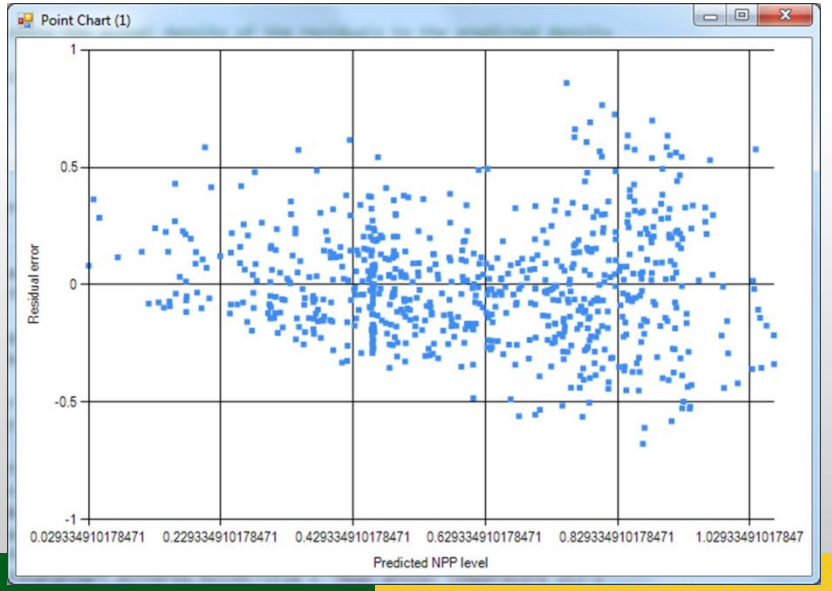



```
let mx k = NwayMixture.M(VectorGaussian.M,k)
let M2 = Evidence.M(mx 3, mx 6)
```

Fitting Model to Climate Data (TACAS'13)

- We developed scientific models as Fun models
- One benefit is the automatic extraction of the likelihood function as the density of a probabilistic expression






```
module NPP =
  let predict w x =
    let prec_lim = w.max_NPP * (1.0 - exp (-w.p * x.MAP))
    let temp_lim = w.max_NPP / (1.0 + exp (w.t1 - w.t2 * x.MAT))
    let pred_NPP = min prec_lim temp_lim
    pred_NPP

  let model =
   {Prior =
      <@ fun () ->
         {max_NPP = random(Gamma(1.0, 1.0))
          p       = random(Gamma(1.0, 1.0))
          t1      = random(Gamma(1.0, 1.0))
          t2      = random(Gamma(1.0, 1.0))
          s_NPP   = random(Gamma(1.0, 1.0))} @>
    Gen =
      <@ fun (w,x) ->
         {NPP = random(Gaussian(predict w x,
                                w.s_NPP * w.s_NPP))} @>}
```

Infer.NET Fun

- Bayesian inference by functional programming
  - Write your model in F#
  - Run forwards to synthesize data (normal F#)
  - Run backwards to infer parameters (via Infer.NET)
- Benefits:
  - Models are simply code in F#'s simple succinct syntax
  - Higher-level features than core Infer.NET: tuples, records, array comprehensions, and functions
- A wide range of efficient algorithms for regression, classification, and specialist learning tasks derive by probabilistic functional programming.
- Papers, download available: http://research.microsoft.com/fun

Challenges

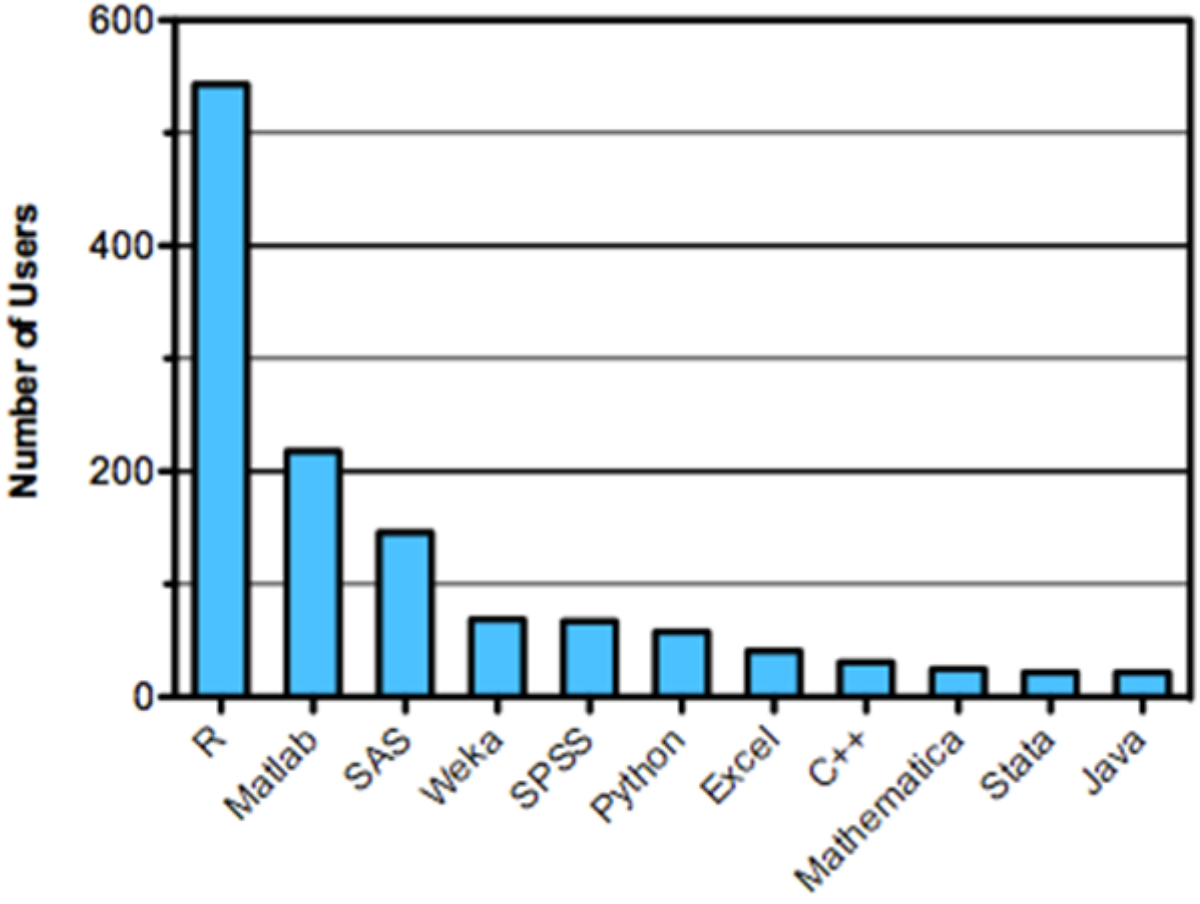
Three Challenges

- Poor usability could be a show-stopper
- Fragmentation
- Potential beneficiaries may not have the time, inclination, or aptitude to learn to write and debug probabilistic programs.

Pain Points of Probabilistic Programming

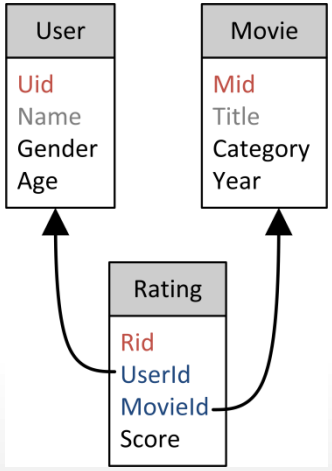
- **15%:** "***Complicated object model*** in language/library syntax and type system."
- **15%:** "***Gap between declarations and operational semantics***."
  - "You can write graphical models that make sense but can't execute due to internal details of the engines."
- **20%:** "***Tuning is time-consuming*** (parameters/algorithm selection, no. of iterations)."
  - "I spent most of my time on robustness; setting hyperparameters and the priors ."
- **20%:** "***Performance*** (cost of model in memory, perf impact of designs), scalability."
  - "It would be nice if a simple annotation could inform the model of how to batch elements."
- **30%:** "***Understanding inference results is hard***."
  - "Once you have a model running, there's no explanation for the inference, hard to find whether issues come from modelling, features, parameters, or the data deficiencies."

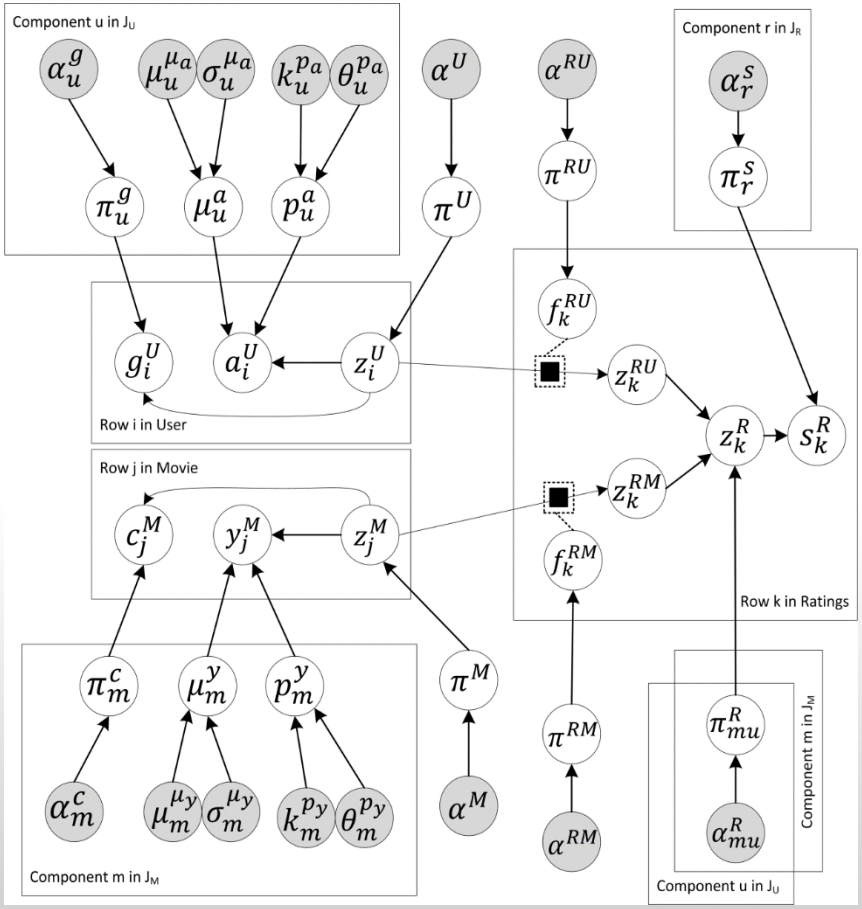
Is there better data? Should we gather more to create a baseline?


Number of Users
600
400
200
0
R
Matlab
SAS
Weka
SPSS
Python
Excel
C++
Mathematica
Stata
Java

Probabilistic Metaprogramming

- Singh and Graepel's InfernoDB





Questions?