

Appendix of Paper “Traffic Engineering with Forward Fault Correction”

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1. INTRODUCTION

In our recently published paper [1], we propose a method which leverages bubble sorting network to transfer “Bounded M-sum constraint” into an efficient format. In Section 4.4.2, we show in Figure 8(c) that we use linear formulations to encode the constraints in lines 10 and 11 of Algorithm 2. Nevertheless, this encoding is not precise. The linear constraints in Figure 8(c) is only approximating the original constraints with absolute value formats in Algorithm 2.

In this technical report, we will explain why our bubble sorting network method and FFC implementation are valid despite of the preceding imprecision of the linear formulation. We will prove that the linear formulation shown in Figure 8(c) is sufficient to guarantee that the final TE this formulation computes satisfies all FFC requirements.

2. BOUNDED M-SUM CONSTRAINT

In section 4.4, we show that FFC constraints on both control and data plane can be transferred into a single constraint format, which is called “Bounded M -sum constraint”.

DEFINITION 1 (BOUNDED M -SUM CONSTRAINT). *Given a set of variables $X = \{x_1, \dots, x_n\}$, and let $x^{(i)}$ be the i th largest element in X , this constraint requires that:*

$$\sum_{i=1}^M x^{(i)} \leq B \quad (1)$$

where B is a bound.

In other words, if we can make sure that (1) is true in the final TE solution, the TE solution then is surely to satisfy the FFC requirements.

In section 4.4.2, we show how to compress the number of constraints in (1) from $O(\binom{n}{M})$ to $O(kn)$ with a bubble sorting network.

3. THE IMPRECISION IN FORMULATION

Let $Y = \{y_1, \dots, y_n\}$ be the output of a complete bubble sorting network. In section 4.4.2, we claim that $y_i = x^{(i)}$. This is true ideally if we use the comparison suggested by Algorithm 2.

$$\begin{aligned} x_{max} &= \frac{x_1 + x_2 + |x_1 - x_2|}{2} \\ x_{min} &= \frac{x_1 + x_2 - |x_1 - x_2|}{2} \end{aligned} \quad (2)$$

However, in practice we need to encode (2) with linear constraints. Therefore, in Figure 8 (c), we show that in our implementation we use the following linear constraints to approximate (2):

$$\begin{aligned} x_{max} &= \frac{x_1 + x_2 + d}{2} \\ x_{min} &= \frac{x_1 + x_2 - d}{2} \\ -d &\leq x_1 - x_2 \leq d \\ d &\geq 0 \end{aligned} \quad (3)$$

Because of this approximation, the statement that $y_i = x^{(i)}$ does not hold anymore. Nevertheless, the following property still holds:

$$\forall k \in \{1, \dots, n\} : \sum_{i=1}^k y_i \geq \sum_{i=1}^k x^{(i)} \quad (4)$$

This means that $\sum_{i=1}^M y_i$ is an upper-bound of the maximum sum of M elements in X . By ensuring $\sum_{i=1}^M y_i \leq B$, we still achieve the guarantee in (1). Hence, the approximation in (3) still makes sure that the final TE found via the bubble sorting network method satisfies all FFC requirements.

4. THE CORRECTNESS OF BUBBLE SORTING NETWORK METHOD

Next, we prove that (4) always holds.

After the m th round of bubble, we get y_m and a collection of residual variables $X_m = \{x_{m,1}, \dots, x_{m,n-m}\}$. Denote $x_m^{(i)}$ as the i th largest element in X_m . Firstly, we have following lemma:

LEMMA 1. *After the m th round of bubble, the following inequation always holds:*

$$\forall k \in \{1, \dots, n-m\} : y_m + \sum_{i=1}^k x_m^{(i)} \geq \sum_{i=1}^{k+1} x_{m-1}^{(i)} \quad (5)$$

PROOF. Let π is an arbitrary permutation of nature numbers $\{1, \dots, k+1\}$, and $\pi(i)$ is the i th element in π . Suppose that in the m th round bubble, $x_{m-1}^{(1)}, \dots, x_{m-1}^{(k+1)}$ are compared with the order indicated by π .

At the comparison with $x_{m-1}^{(\pi(i))}$, denote $v_{m-1}^{(i)}$ as the other input of the comparison, and $\bar{v}_{m-1}^{(i)}$ and $w_{m-1}^{(i)}$ as the larger

and smaller output of this comparison. From (3) we can see that:

$$\forall i : x_{m-1}^{(i)} + v_{m-1}^{(i)} = \bar{v}_{m-1}^{(i)} + w_{m-1}^{(i)} \quad (6)$$

At the comparison of $x_{m-1}^{(\pi(1))}$, we have:

$$\bar{v}_{m-1}^{(\pi(1))} \geq x_{m-1}^{(\pi(1))} \quad (7)$$

At the comparison of $x_{m-1}^{(\pi(i))}$, $2 \leq i \leq k+1$, we have:

$$\forall i \in [2, k+1] : \bar{v}_{m-1}^{(\pi(i))} + w_{m-1}^{(\pi(i))} = x_{m-1}^{(\pi(i))} + v_{m-1}^{(\pi(i))} \quad (8)$$

Additionally, because of the structure of bubble sorting, we also have:

$$\forall i \in [2, k+1] : v_{m-1}^{(\pi(i))} \geq \bar{v}_{m-1}^{(\pi(i-1))} \quad (9)$$

Therefore, we derive:

$$\forall i \in [2, k+1] : \bar{v}_{m-1}^{(\pi(i))} + w_{m-1}^{(\pi(i))} \geq x_{m-1}^{(\pi(i))} + \bar{v}_{m-1}^{(\pi(i-1))} \quad (10)$$

If we make a sum of (7) and (10), we derive:

$$\bar{v}_{m-1}^{(\pi(k+1))} + \sum_{i=2}^{k+1} w_{m-1}^{(\pi(i))} \geq \sum_{i=1}^{k+1} x_{m-1}^{(\pi(i))} = \sum_{i=1}^{k+1} x_{m-1}^{(i)} \quad (11)$$

Because $w_{m-1}^{(i)}$, $\forall i \in [1, k+1]$ are all elements in X_m respectively, from (10), we have:

$$y_m + \sum_{i=1}^k x_m^{(i)} \geq \bar{v}_{m-1}^{(\pi(k+1))} + \sum_{i=2}^{k+1} w_{m-1}^{(\pi(i))} \geq \sum_{i=1}^{k+1} x_{m-1}^{(i)} \quad (12)$$

, which finishes the proof. \square

THEOREM 1. *Given n variables $X = \{x_1, \dots, x_n\}$ and $x^{(i)}$ as the i th largest element in X , and let $Y = \{y_1, \dots, y_n\}$ as the output of bubble sorting network which encodes each comparison with (3), we always have:*

$$\forall m : \sum_{i=1}^m y_i \geq \sum_{i=1}^m x^{(i)} \quad (13)$$

PROOF. From Lemma 1, we know:

$$\begin{aligned} \forall m : \sum_{i=1}^m x^{(i)} &\leq y_1 + \sum_{i=1}^{m-1} x_1^{(i)} \\ &\leq y_1 + y_2 + \sum_{i=1}^{m-2} x_2^{(i)} \leq \dots \leq \sum_{i=1}^m y_i \end{aligned} \quad (14)$$

, which finishes the proof. \square

5. REFERENCES

- [1] H. H. Liu, S. Kandula, R. Mahajan, M. Zhang, and D. Gelernter, "Traffic Engineering with Forward Fault Correction," in *SIGCOMM'14*.