ROBUST DESIGN OF WIDEBAND LOUDSPEAKER ARRAYS

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ABSTRACT

Loudspeaker arrays usually are used in professional sound reinforcement systems to provide uniform sound coverage of the listening area. They can also be used for focusing the sound to the user for semi-private communication and reducing the overall noise pollution. In this paper we describe a procedure for designing broadband beamformers for loudspeaker arrays that is robust to the manufacturing tolerances of the loudspeakers – the limiting factor for achieving high directivity. The designed beamformer is evaluated using simulations and measurements of actual loudspeaker array.

***Index Terms* —** Acoustic arrays, Acoustic beam focusing, Robustness, Audio systems, Tolerance analysis.

1. INTRODUCTION

Arrays of closely spaced loudspeakers have been around for decades. Their increased directivity is used to achieve better sound in reverberant environments [6]. Later it was proven that for a given source position the sound field in a hall can be acquired with full temporal and spatial information [1] and then reproduced using multiple loudspeakers by wave field synthesis [2]. The potential applications for loudspeaker arrays include compensating for the room response, focusing the sound in a small region and virtual reality systems [4].

One of the first attempts to create a theory and build a working prototype of a loudspeaker array with a controllable directivity pattern was presented in [5]. The proposed prototype uses a set of bandpass filters, a series of hardware delay lines and analog amplifiers with different gain to form the loudspeaker array directivity pattern. Whereas that design was limited by the available hardware, today's computers allow real-time implementation with many long linear filters.

The design of the directivity pattern of arrays of transducers is well studied for the case of antenna arrays, as they are widely used for building the wireless network for cellular phones [7]. Many algorithms, designed for this purpose, were later borrowed for the design of microphone arrays and loudspeaker arrays. Designing arrays of transducers for audio is more complex due to the wideband character of the audible sound: difference in wavelength tens and hundreds of times within the working bandwidth; more complex directivity patterns of the transducers (microphones and loudspeakers) and higher tolerances in their parameters [8].

A method for array pattern synthesis using semidefinite programming is proposed in [7]. The authors explore several criteria for array optimality in the design of a narrowband antenna array. The second part of the paper includes directivity pattern synthesis robust to gain and phase uncertainty. The authors of [8] build a mathematical model of a microphone array directivity pattern, accounting for the tolerances of the microphone parameters. They use a direct optimization method, large number of microphone array instances, and various optimization criteria to design a beamformer for an array of omnidirectional microphones.

In this paper we present a beamformer design algorithm for wideband loudspeaker arrays that is robust to transducer tolerances. We discuss the scenario in which the desired goal is to focus the sound in one area, and reduce the audibility in the rest, but the approach and the results are applicable for other design goals. The loudspeakers' directivity patterns and their variations are included in the model. We explore various optimization criteria to design the beamformer. The proposed method is illustrated with the design of a 16-element linear loudspeaker array for focusing the sound in a particular area. Simulation and experimental results are presented at the end of the paper.

2. MODELLING

Consider an array of M loudspeakers with known positions, determined by vector $\vec{p}$; the loudspeakers emit sound at locations $p_m = (x_m, y_m, z_m) : m = 0,1,\cdots,M-1$. The design of the array and the array processing will be done in the frequency domain. Under the assumption of independent frequency bins we perform the design in each frequency bin and therefore frequency bin indices will be omitted for simplicity whenever it is possible.

Each loudspeaker has a known directivity pattern $U_m(f,\theta)$, where θ is the incident angle. Here we assume symmetric spatial response around the main response axis at 0^O. The loudspeaker directivity pattern is a complex function, providing the spatio-temporal transfer function of this channel. For an ideal omnidirectional loudspeaker $U_m(f,\theta) = \text{constant}$. In some cases the loudspeaker array

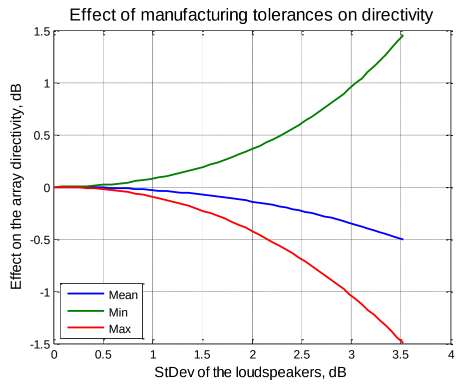

Figure 1. Effect of manufacturing tolerances on the loudspeaker array directivity.

can have speakers of different types, so $U_m(f,\theta)$ can vary with m. In addition, the loudspeakers have manufacturing tolerances, which makes the directivity pattern vary for the same type of speakers. Let's assume that the probability distribution is Gaussian with mean $\bar{U}_m(f,\theta)$ and known variation $\sigma^2(f)$, independent of the incident angle. The manufacturing tolerances can be modeled as zero mean complex noise added to the averaged directivity pattern:

$$U_m(f,\theta) = \bar{U}_m(f,\theta) + \mathbb{N}\left(0,\sigma^2(f)\right) \tag{1}$$

The sounds from all speakers are superimposed in the discrete listening volume, for each point $v_s = (x_s, y_s, z_s)$ forming signal

$$X_s(f) = \sum_{i=0}^{M-1} \frac{\exp\left(-j2\pi f \left\|p_i - v_s\right\|/c\right)}{\left\|p_i - v_s\right\|}.U_i\left(f, \sphericalangle(p_i, v_s)\right).Y_i(f) \tag{2}$$

or

$$X_s(f) = \sum_{i=0}^{M-1} D_i(f, v_s).U_i\left(f, \sphericalangle(p_i, v_s)\right).Y_i(f), \tag{3}$$

where $D_i(f,v_s) = \dfrac{\exp(-j2\pi f \left\|v_i - p_s\right\|/c)}{\left\|v_i - p_s\right\|}$ is defined for each point of the listening volume. Note that here we ignore the mutual radiation impedance effects as they are significant for frequencies below our work diapason [9]. The L listening points can be uniformly or randomly placed in the listening volume with distance between them less than half of a wave length $\lambda = c/f$. They should not be placed close to the loudspeakers of the array. $Y_i(f)$ is the sound emitted from each speaker, which is the same sound $S(f)$ filtered by N-tap time-invariant and data-independent filter bank, represented in frequency domain by vector $\mathbf{W}$ with length M for each of the K frequency bins. This finalizes the sound field forming equation as

$$X_s(f, v_s) = \mathbf{W}^T(f)\aleph(f, v_s)S(f) \tag{4}$$

where $\aleph(f, v_s) = \mathbf{D}(f, v_s)\circ\mathbf{U}(f, v_s)$ is the sound propagation vector and $\circ$ denotes Hadamard product.

Once we have the sound propagation equation the directivity ratio can be computed as the proportion of the average power in the listening area and the average power in the silent areas:

$$\mathbb{R} = \frac{L}{L_A}\frac{\sum_{s\in A}\left|\mathbf{W}^T\left(\mathbf{D}_s\circ\mathbf{U}_s\right)\right|^2}{\sum_{s}\left|\mathbf{W}^T\left(\mathbf{D}_s\circ\mathbf{U}_s\right)\right|^2}, \tag{5}$$

where A denotes the listening area and L_A is the number of points in the listening area.

The designed beamformer will require emitting certain power from each of the loudspeakers. The total power is proportional to the sum of the weights squares:

$$P_{tot} \approx \mathbf{W}^T\mathbf{W} = \sum_{i=0}^{M-1}\left|W_i\right|^2. \tag{6}$$

3. DIRECTIVITY AND TOLERANCES

Combining the loudspeaker model with tolerances (1) with directivity equation (5) we have:

$$\tilde{\mathbb{R}} = \frac{L}{L_A}\frac{\sum_{s\in A}\left|\sum_{i=1\div M} W_i D_{si}\left(\bar{U}_{si} + \mathbb{N}_i\left(0,\sigma^2\right)\right)\right|^2}{\sum_{s}\left|\sum_{i=1\div M} W_i D_{si}\left(\bar{U}_{si} + \mathbb{N}_i\left(0,\sigma^2\right)\right)\right|^2} \tag{7}$$

and after some transformations we can estimate the average directivity:

$$\bar{\mathbb{R}} = \mathbb{R}\frac{1}{1+\dfrac{\sigma^2}{2\left|\bar{R}_S\right|}\sqrt{P_{tot}}} + \frac{\dfrac{\sigma^2}{2\left|\bar{R}_A\right|}\sqrt{P_{tot}}}{1+\dfrac{\sigma^2}{2\left|\bar{R}_S\right|}\sqrt{P_{tot}}}. \tag{8}$$

Here $\bar{R}_A$ and $\bar{R}_S$ are the average distances from the center of the loudspeaker array to the listening and silent areas respectively. Indexes for frequency are omitted for clarity. As $\mathbb{R} > 1$ we will have degradation in the average directivity when transducer mismatch is present. If we assume that the Gaussian distribution of the manufacturing tolerances is pruned to $\pm 2.5\sigma$, i.e. if the quality control at the manufacturer removed the transducers which differ too much from the specifications, we can find limiting estimate for the worst directivity:

$$\mathbb{R}_{\min} = \mathbb{R}\frac{1}{1+\dfrac{(2.5\sigma)^2}{2\left|\bar{R}_S\right|}\sqrt{P_{tot}}} - \frac{\dfrac{(2.5\sigma)^2}{2\left|\bar{R}_A\right|}\sqrt{P_{tot}}}{1+\dfrac{(2.5\sigma)^2}{2\left|\bar{R}_S\right|}\sqrt{P_{tot}}} \tag{9}$$

In a similar way can be estimated the upper limit of the directivity. Figure 1 shows the effect of the channel mismatch on the loudspeaker array directivity. The results are similar to microphone array sensitivity analysis, described in [10].

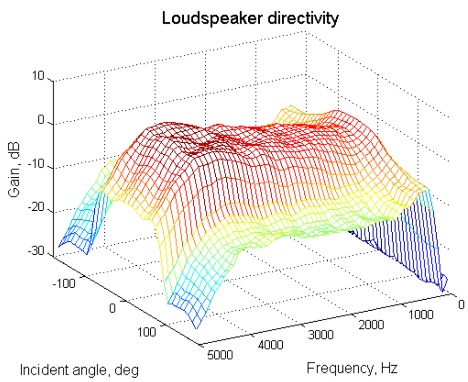

Figure 2. Loudspeaker magnitude response.

4. BEAMFORMER DESIGN

To design the loudspeaker array beamformer means to estimate the weights vectors **W**. The solution usually minimizes certain goal function under given constrains, i.e. is optimal in one or another way. The target directivity pattern $T(v_s)$ gives the desired shape of the formed sound field. For the case when we want to focus the sound from the speaker array in certain area A the target pattern can be defined as

$$T(v_s)=\begin{vmatrix}1 & v_s\in \mathrm{A}\\ 0 & otherwise\end{vmatrix} \quad (10)$$

Of course, the target pattern definition can be more complex than (10). Frequently there are zones in the sound field we do not care about, usually the transition between audible/non audible zones. Additional areas are the space around each of the loudspeakers. This includes the volume of the speaker array itself and nearby area where the model of propagation of the sound wave in free space is not adequate. In addition, due to the $1/r$ sound intensity propagation law, around speakers we have poles in the propagation we may want to avoid during the design and simulations. These areas can be described by assigning weights to the regions in the target pattern, defined as $T_W(v_s)$. A simple definition of this target pattern weight is:

$$T_W(v_s)=\begin{vmatrix}w_A & v_s\in A, & v_s\notin C\\ w_S & v_s\notin A, & v_s\notin C\\ 0 & v_s\in C\end{vmatrix} \quad (11)$$

where C is a don't care region, w_A is the weight we put to the places where the speaker array should be audible and w_S is the weight we put to the places where the sound should be suppressed. In the simplest case $w_A=w_S=1$.

The loudspeaker array beamformer design should meet certain constrains. One of the typical requirements is to have equalized power response and average zero phase shift in the listening area. Both magnitude and phase equalization are relative and need a reference to equalize to – this can be the sound field generated from an omnidirectional loudspeaker in the center of the array $O(f,v_i)$. Then the response normalization constrain is defined as:

$$Q_N=\sum_{\forall v_i\in A}\left|X(f,v_i)-O(f,v_i)\right|^2\Rightarrow 0 \quad \forall f\in\left[f_{\min},f_{\max}\right] \quad (12)$$

In addition we should not overload the amplifiers and loudspeakers, which means that the power constrain is:

$$Q_{Pi}=\left|W_i(f)\right|\le 1.0 \quad \forall f\in\left[f_{\min},f_{\max}\right] \quad i=1\div M \quad (13)$$

Converting the constrained optimization problem to non-constrained is done by changing the optimization goal to:

$$C_{NC}=C_C+Q_N+\sum_{i=1\div M}\min\left[0,\left(Q_{Pi}-1\right)\right] \quad (14)$$

Here C_{NC} is the non-constrained optimization goal and C_C is the constrained. As optimization goal we can use:

- maximum directivity $C_{MaxDir}=-\mathbb{R}$, computed from (5) with using the weighting, defined in (11);
- robust maximum directivity $C_{RobMaxDir}=-\mathbb{R}_{\min}$, i.e. to maximize the weighted with (11) equation (9);
- minimum output power $C_{MinPower}=P_{tot}$, from (6), when causing the same audio field intensity in the listening area.

Table 1. Results for three optimization goals.

Goal	Power	simDir	avDir	stdDir	minDir
MaxDir	1.53	21.40	19.52	1.01	17.41
RobMaxDir	-11.01	21.24	21.06	0.38	20.15
MinPower	-11.07	20.48	20.41	0.35	19.56

5. EXPERIMENTAL RESULTS

Experiments were performed using the design approach above with an array of 16 equally spaced loudspeakers with spacing distance of 125 mm. The loudspeakers were placed in wooden enclosure with proper separation and damping inside. The listening area was defined as circle with radius 0.2 m positioned 1.5 m in front of the array, which . From array processing perspective this is the worst directivity position for broadband arrays.

The loudspeaker model was measured in an anechoic chamber by rotating the speaker array and playing wideband chirp signal from each speaker separately. A microphone placed 1.5 m from the center of the array was used as a sensor. The final model $U(f,\theta)$ was obtained by averaging the individual models and the values for $\sigma(f)$ were computed using the 16 individual models. We can obtain the model value for any frequency and direction using interpolation between the values in these discrete models. The shape of the loudspeaker magnitude response as function of the incident angle and frequency is shown on figure 2.

The beamformer design was performed using a gradient descent optimization algorithm and the unconstrained goal (14) for the three constrained optimization goals, specified

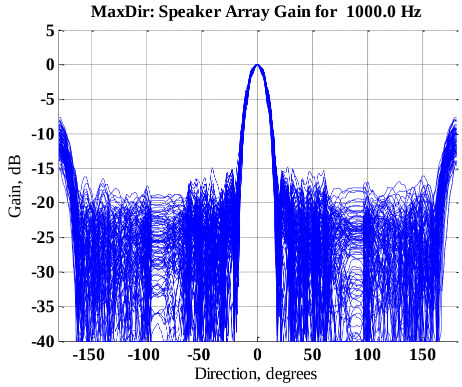

Figure 3. Maximum directivity goal – directivity patterns.

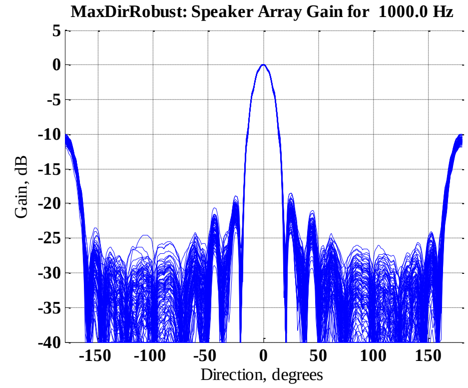

Figure 4. Robust maximum directivity goal – directivity patterns.

in previous section: maximum directivity, robust maximum directivity, and minimum power.

Each of the three solutions was evaluated under perfect channel matching conditions and with 100 instances of 16 element arrays with loudspeakers with random parameters, according to the distribution, specified in the second section. Table 1 presents the results for 1000 Hz: total power, simulated directivity with channel match, and average, standard deviation and minimal directivity when we have channel mismatch. All numbers are in dB, the power reference is a single omnidirectional speaker. Figure 3 shows the directivity patterns for distance of 1.5 m for all 100 random instances of the loudspeaker array for weights, computed with maximum directivity criterion. Figure 4 shows the same for the weights, computed using robust maximum directivity criterion. The difference is well visible; the second set of weights produces better directivity with the same variations in the channel mismatch. Note the substantially lower power for producing the same sound field intensity in the listening area, combined with better directivity for the robust directivity goal. Figure 5 shows the sound field intensity in an area 5 m x 5 m with loudspeaker array in the center. The better directivity, provided by the second beamformer was confirmed with direct measurements in anechoic chamber.

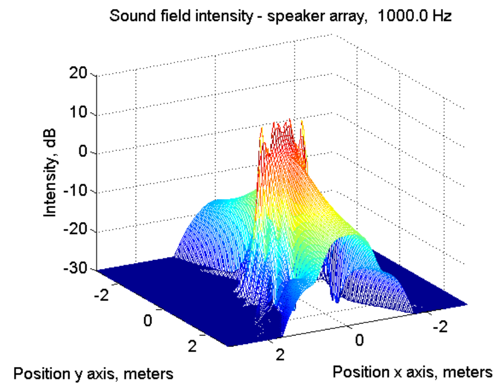

Figure 5. Sound field intensity for 1000 Hz.

6. DISCUSSION AND CONCLUSION

The proposed approach for designing wideband beamformers for loudspeaker arrays provide solutions that are robust to manufacturing tolerances. The agreement between modeling and experimental results is good. The analytical solution for the minimum directivity (9) allows the weights to be computed without computationally demanding statistical approaches. This approach is applicable for any target pattern when designing beamformers for arrays of transducers.

7. REFERENCES

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