Checking Compatibility of Bit Sizes in Floating Point Comparison Operations

Manuel Fähndrich[1]

Microsoft Research, Redmond

Francesco Logozzo[2]

Microsoft Research, Redmond

Abstract

We motivate, define and design a simple static analysis to check that comparisons of floating point values use compatible bit widths and thus compatible precision ranges. Precision mismatches arise due to the difference in bit widths of processor internal floating point registers (typically 80 or 64 bits) and their corresponding widths when stored in memory (64 or 32 bits). The analysis guarantees that floating point values from memory (*i.e.* array elements, instance and static fields) are not compared against floating point numbers in registers (*i.e.* arguments or locals).
Without such an analysis, static symbolic verification is unsound and hence may report false negatives.
The static analysis is fully implemented in Clousot, our static contract checker based on abstract interpretation.

Keywords: Abstract Interpretation, Design by Contracts, Floating points, Numerical Abstract Domains, Static Analysis, .NET.

1 Introduction

Comparing floating point values in programs can introduce subtle errors due to precision mismatches of the compared values. Precision mismatches arise as a result of truncating typically larger register internal floating point widths (80 or 64 bits) to the floating point width used when storing the value into main memory (64 or 32 bits). Such mismatches may produce unexpected program

[1] Email:maf@microsoft.com

[2] Email:logozzo@microsoft.com

```
public class Account
{
  private float balance;

  public Account(float initial)
  {
    Contract.Requires(initial >= 0.0f);

    balance = initial;
  }

  public void Deposit(float amount)
  {
    Contract.Requires(amount >= 0.0);
    Contract.Ensures(balance == Contract.OldValue(balance) + amount);

    balance = balance + amount;
  }

  // Other methods here
}
```

Fig. 1. A C# code snippet for the classical Bank account example. `Contract.Requires` specifies the precondition, `Contract.Ensures` specify the postcondition, `Contract.OldValue` denotes the value of the argument expression at the entry point (not directly expressible in C#). It turns out that, as it is, the postcondition is incorrect, and it may fail at runtime for opportune values of `amount`.

behavior, resulting in programmer confusion and—if ignored—unsound static program analysis.

We introduce the problem with the code snippet in Fig. 1, extracted from the *"classical"* bank account example annotated with contracts [8]. In this paper, we use C# as our language and the .NET runtime. However, the general problem addressed in this paper is present in numerous programming languages and runtimes. We address these other contexts in Sect. 5.

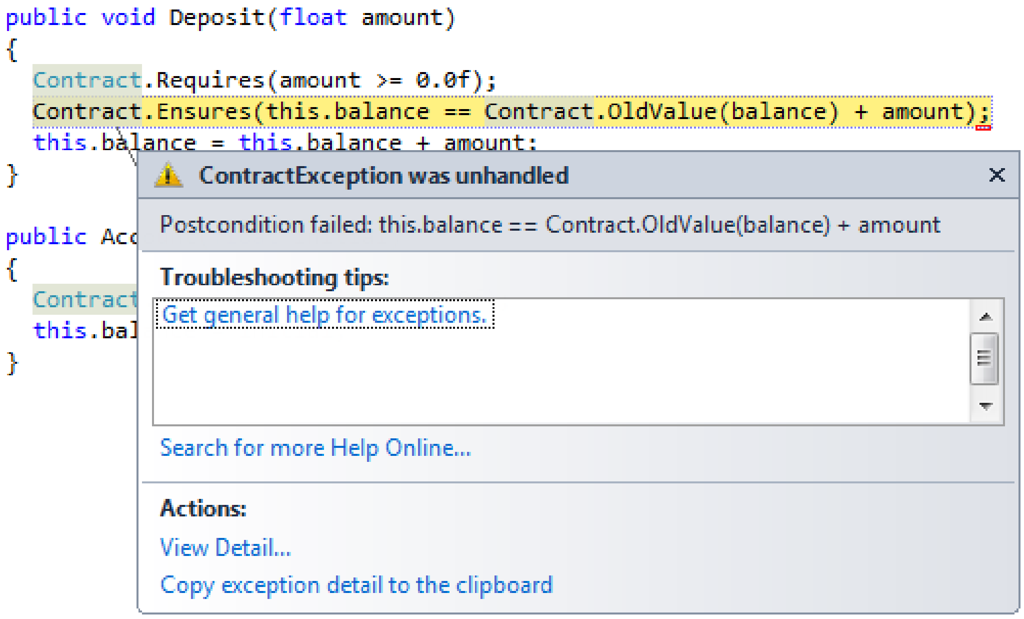
The class `Account` represents a bank account. The method `Deposit` updates the balance by a given non-negative `amount`. The postcondition for `Deposit` states that on method exit the balance has been correctly updated. The current balance is stored in an instance field of type `float`. The ECMA standard requires `.NET` implementations of floating point types to follow the IEC:60559:1989 standard.

At a first glance, one expects the postcondition to hold and any static analyzer to easily prove it. In fact, a simple reasoning by symbolic propagation ($\mathtt{balance}_0$ denotes the value of the field `balance` at method entry) could be:

$$\begin{array}{ll}
 & \mathtt{assert}\ \mathtt{balance} == \mathtt{balance}_0 + \mathtt{amount} \\
\Longleftrightarrow & \{ \text{ by the assignment: } \mathtt{balance} = \mathtt{balance}_0 + \mathtt{amount} \ \} \\
 & \mathtt{assert}\ \mathtt{balance}_0 + \mathtt{amount} == \mathtt{balance}_0 + \mathtt{amount} \\
\Longleftrightarrow & \{ \text{ by equality } \} \\
 & \mathtt{true}
\end{array}$$

Unfortunately, a static analyzer for `.NET` performing this reasoning would be unsound! For instance, the following two lines of `C#` code:

```
var account = new Account(6.28318548f);
account.Deposit(3.14159274f);
```



```
public void Deposit(float amount)
{
  Contract.Requires(amount >= 0.0f);
  Contract.Ensures(this.balance == Contract.OldValue(balance) + amount);
  this.balance = this.balance + amount;
}

public Acc
{
  Contract
  this.bal
}
```

ContractException was unhandled

Postcondition failed: this.balance == Contract.OldValue(balance) + amount

Troubleshooting tips:
Get general help for exceptions.
Search for more Help Online...
Actions:
View Detail...
Copy exception detail to the clipboard

Fig. 2. A failure at runtime of the postcondition for `Deposit`.

cause a postcondition violation in the method `Deposit` (cf. Fig. 2).
What is wrong here? Let's first rule out causes that are *not* the problem:

- Overflow can be excluded, as floating point numbers cannot overflow (at worst, operations result in special values ±∞ or `NaN`).
- Non-determinism is ruled out by the IEEE754 standard, and by the fact that the code in the example is single-threaded.
- Cancellation is to be ruled out too: *e.g.* the numerical quantities are positive and of the same order of magnitude.
- Floating point addition is commutative, so this is not the cause of the problem either.
- Addition is *not* necessarily associative, but we do not need associativity here (we are adding only two numbers).

The real culprit here is the equality test. In general all comparisons of floating point values are problematic. However it is still unclear at first sight why the comparison is a source of problems here: after all we are adding up the same two quantities and then comparing them for equality. If some rounding error occurs, then the same error should occur in both additions, or won't it?

The reason for the unexpected behavior is to be found deeper in the specification of the Common Language Runtime (Partition I, Sect. 12.1.3 of [1]):

> *Storage locations for floating-point numbers (statics, array elements, and fields of classes) are of fixed size. The supported storage sizes are* `float32` *and* `float64` *. Everywhere else (on the evaluation stack, as arguments, as return types, and as local variables)* ***floating-point numbers are represented using an internal floating-point type****. In each such instance, the*

$\mathtt{l} = k$	*(load const)*		
$\mathtt{l} = \mathtt{l}_1$	*(copy)*	$\mathtt{l} = (\mathtt{type})\ \mathtt{l}_1$	*(cast)*
$\mathtt{l} = \mathtt{l}_1\ op\ \mathtt{l}_2$	*(binary op)*		
$\mathtt{l} = \mathtt{o.f}$	*(load field)*	$\mathtt{o.f} = \mathtt{l}$	*(store field)*
$\mathtt{l} = \mathtt{a}[\mathtt{l}_\mathtt{i}]$	*(load array)*	$\mathtt{a}[\mathtt{l}_\mathtt{i}] = \mathtt{l}$	*(store array)*

Fig. 3. The simplified bytecode language we consider for the analysis. A constant k can only be a constant belonging to the `float32` or `float64` ranges. Casting is allowed only to $\mathtt{type} \in \{\mathtt{float32}\ , \mathtt{float64}\ \}$.

> *nominal type of the variable or expression is either* `float32` *or* `float64` *, but its value can be represented internally with additional range and/or precision. The size of the internal floating-point representation is implementation-dependent, can vary, and shall have precision at least as great as that of the variable or expression being represented. An implicit widening conversion to the internal representation from* `float32` *or* `float64` *is performed when those types are loaded from storage. [...]* ***When a floating-point value whose internal representation has greater range and/or precision than its nominal type is put in a storage location, it is automatically coerced to the type of the storage location****.*

The standard allows exploiting the maximum precision available from the floating point hardware for operations on values in registers *despite* of their nominal type, *provided* that on memory stores the internal value is truncated to the nominal size. It is now easy to see why we get the postcondition violation at runtime.

The result of the evaluation of the expression $\mathtt{this.balance} + \mathtt{amount}$ is internally stored at the maximum precision available from the hardware (on Intel processors 80 bits registers, on ARM architectures 64 bits). In the example, the result of the addition is 9.42477822, a value that cannot be precisely represented in 32 bits.

The successive field store forces the value to be truncated to 32 bits, thereby changing the value. In the example, 9.42477822 is coerced to a `float`, causing a loss of precision resulting in the value 9.424778 being stored in the field `this.balance`.

When the postcondition is evaluated, the truncated value of `balance` is re-loaded from memory, but the addition in the postcondition is re-computed with the internal precision. Comparing these two values causes the postcondition to fail, since $9.424778 \neq 9.42477822$.

Contribution

We present a simple static analysis to check that floating point comparisons (equalities, inequalities) use operands of *compatible* types. When they are not compatible, the analysis reports a warning message to the user, so that all successive validations should be understood as conditional. We fully implemented the analysis in Clousot, our static contract checker based on abstract interpretation for .NET [2]. We validated the analysis by running it on the base class library of .NET where it emitted 5 real warnings.

2 The Language

We illustrate our analysis on a minimalistic bytecode language. We make some simplyfing assumptions. There are two kinds of variables: store variables ($\mathtt{f}, \mathtt{a} \in \mathtt{S}$) and locals ($\mathtt{l}, \mathtt{p} \in \mathtt{L}$). *Store* variables are instance fields, static fields and arrays. *Local* variables are locals, parameters, and the return value. Variables belong to the set $\mathtt{Vars} = \mathtt{S} \cup \mathtt{L}$. Aliasing is not allowed.

The language has only two nominal floating point types ($\mathtt{float32}$,$\mathtt{float64}$ $\in \mathtt{T_N}$) and one internal floating point type ($\mathtt{floatX}$) such that $64 \leq \mathtt{X}$. On x86, floatX is float80, allowing extended floating point precision. Please note that the .NET standard does not include a long double as for instance C [3], so application programmers have no access to floatX types.

All variables have a nominal floating point type. At runtime, the nominal floating point type for locals may be widened but not that of store variables. We say "may be widened", as it depends on register allocation choices by the compiler. We think it is reasonable to force the code to compute values independent of floating point register allocation choices.

The simplified bytecode language is presented in Fig. 3. Floating point constants are loaded into locals *(load const)*. Admissible constant values are only those admitted by the nominal types, *i.e.* floating point constants in 32 or 64 bits including special values as $\pm\infty$ and NaN (Not-a-number).

Values can be copied to locals, retaining their internal value *(copy)*. Casting is allowed only to nominal types, with values narrowed or widened as needed *(cast)*. In general it is not true that if $\mathtt{l_1}$ and $\mathtt{l}$ have the same *nominal* type then *(cast)* is semantically equivalent to *(copy)* as their *internal* type may differ.

Binary operations are the usual floating point arithmetic ones $(+, -, *, /)$ and (unordered) comparison operations $(==, <, \leq,)$ *(binary op)*. The result of a comparison is 0.0 if the comparison is false, 1.0 otherwise.

Values are loaded from and stored to fields *([load/store] field)*. We do not distinguish between static and instance fields. Fields only contain values of nominal types: therefore, when storing a local into a field, its value is automatically narrowed to the field nominal type value. If the value of l is too large or too small, then it is approximated to $\pm\infty$ or to 0. Similarly, values read from arrays have a nominal type value and values written into arrays are narrowed to the nominal type of the array type. Arrays are indexed by local values, and in addition to the usual out-of-bounds checking, we assume that the computation stops also when $\mathtt{l_i}$ is a floating point number with non-zero decimal part or it is NaN.

Example 2.1 The compilation to simplified bytecode of the body of method deposit (without contracts) of Fig. 1 is in Fig. 4. Please note that the store

$$\begin{array}{lllll}
0: & \mathtt{l}_1 & = & \mathtt{this.balance} \\
1: & \mathtt{l}_2 & = & \mathtt{amount} + \mathtt{l}_1 \\
2: & \mathtt{this.balance} & = & \mathtt{l}_2 \\
3: & \mathtt{l}_1 & = & \mathtt{this.balance} \\
4: & \mathtt{l}_{\mathtt{comp}} & = & \mathtt{l}_1 == \mathtt{l}_2
\end{array}$$

Fig. 4. The (partial) compilation of the running example in our simple bytecode instruction set.

$$\begin{array}{lll}
\llbracket \mathtt{l} = k \rrbracket(\tau) & = & \tau[\mathtt{l} \mapsto \eta(\mathtt{l})] \\
\llbracket \mathtt{l} = \mathtt{l}_1 \rrbracket(\tau) & = & \tau[\mathtt{l} \mapsto \tau(\mathtt{l}_1)] \\
\llbracket \mathtt{l} = (\mathtt{type})\mathtt{l}_1 \rrbracket(\tau) & = & \tau[\mathtt{l} \mapsto \mathtt{type}] \\
\llbracket \mathtt{l} = \mathtt{l}_1\, op \mathtt{l}_2 \rrbracket(\tau) & = & \tau[\mathtt{l} \mapsto \mathtt{floatX}\ ] \\
\llbracket \mathtt{l} = \mathtt{o.f} \rrbracket(\tau) & = & \tau[\mathtt{l} \mapsto \eta(\mathtt{o.f})] \\
\llbracket \mathtt{o.f} = \mathtt{l} \rrbracket(\tau) & = & \tau \\
\llbracket \mathtt{l} = \mathtt{a}[\mathtt{l_i}] \rrbracket(\tau) & = & \tau[\mathtt{l} \mapsto \eta(\mathtt{a}[\cdot]) \\
\llbracket \mathtt{a}[\mathtt{l_i}] = \mathtt{l} \rrbracket(\tau) & = & \tau
\end{array}$$

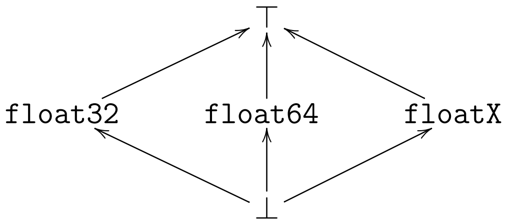
Fig. 5. The abstract semantics. The function η returns the nominal type of a variable.

and load field operations are now made explicit in the bytecode.

3 The Abstract Semantics

3.1 Abstract Domain

The abstract domain $\mathcal{T}$ we use captures the potential runtime floating point width a variable may have, which may be more precise than its nominal type. Therefore, the elements of $\mathcal{T}$ belong to the set $\mathtt{Vars} \longrightarrow \mathtt{T_X}$ where $64 \leq \mathtt{X}$ and $\mathtt{T_X}$ is the abstract domain:



If $\mathtt{X} = 64$, *i.e.* the hardware does not provide any wider floating point register, then `float64` and `floatX` co-incide. This is the case on ARM architectures, but not for `x86` architectures which provide extra precision registers.

The operations of the abstract domain $\mathcal{T}$ (order, join, meet) are the functional pointwise extensions of those on the lattice above. No widening is required as the lattice is of finite height.

3.2 Abstract Semantics

The abstract semantics $[\![\cdot]\!] \in \mathtt{P} \times \mathfrak{T} \longrightarrow \mathfrak{T}$ statically determines, at each program point an internal type for each local variable. Store variables are known, by the ECMA standard, to have their nominal type coincide with the internal type.

The abstract transfer function is defined in Fig. 5. The only constant values that can be explicitly represented are those admissible as `float32` or `float64` values: the internal type of a local after a load constant is its nominal type. Variable copy retains the internal type. The ECMA standard guarantees that casting a value v to `type` truncates the value v to one in the `type` range. If v is too large or too small for `type` then it is rounded to $\pm\infty$ or 0. The result of a binary operation is a value of maximum hardware precision, which we denote by `floatX` . Reading from a field or an array location provides a value of the nominal type (no extra precision can be stored in fields). Writing into a field or an array location causes the truncation of the value to the corresponding nominal type.

Example 3.1 For the bytecode in Fig. 4, with $\tau_0 = [\mathtt{amount} \mapsto \mathtt{floatX}\ ]$, the inferred internal types after each program point are:

$$
\begin{array}{ll}
0: & \tau_1 = \tau_0[\mathtt{l}_1 \mapsto \mathtt{float32}\ ] \\
1: & \tau_2 = \tau_1[\mathtt{l}_2 \mapsto \mathtt{floatX}\ ] \\
2: & \tau_3 = \tau_2 \\
3: & \tau_4 = \tau_3[\mathtt{l}_1 \mapsto \mathtt{float32}\ ] \\
4: & \tau_5 = \tau_4[\mathtt{l}_{\mathtt{comp}} \mapsto \mathtt{floatX}\ ]
\end{array}
$$

3.3 Checking

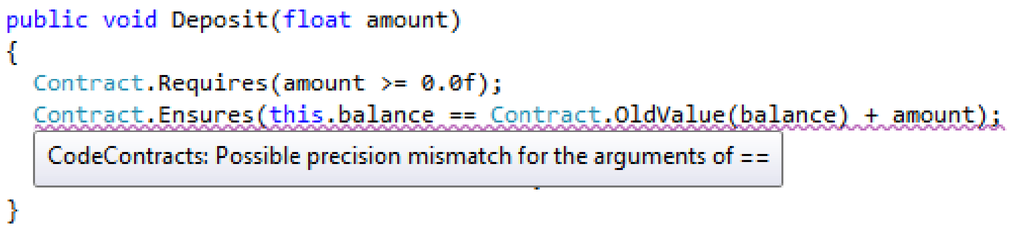
Checking a program `P` for precision mismatch in floating point comparisons is now quite easy. First run the analysis $[\![\mathtt{P}]\!]$ to collect an over-approximation of the internal types for each program point. Then, for each *(binary op)* in `P`

$$\mathtt{pp} : \mathtt{l} = \mathtt{l}_1\ op\ \mathtt{l}_2$$

such that op is one of $==, \leq, <$ get τ_{pp}, which is the abstract pre-state for `pp`.

If $\tau_{pp}(\mathtt{l}_1)$ or $\tau_{pp}(\mathtt{l}_2)$ are different from $\top$, and $\tau_{pp}(\mathtt{l}_1) = \tau_{pp}(\mathtt{l}_2)$ then the comparison is on variables with the same internal type. Otherwise, the comparison may happen on floating point values of different width, and hence a warning to the user should be emitted.

Example 3.2 In our running example, $\tau_5(\mathtt{l}_2) = \mathtt{floatX}$ and $\tau_5(\mathtt{l}_1) = \mathtt{float32}$. So a warning is emitted to the user (cf. Fig 6).



```
public void Deposit(float amount)
{
  Contract.Requires(amount >= 0.0f);
  Contract.Ensures(this.balance == Contract.OldValue(balance) + amount);
```

CodeContracts: Possible precision mismatch for the arguments of ==

```
}
```

Fig. 6. The warning emitted by Clousot.

3.4 Fixing the warnings

When the analysis cannot prove that the operands of a comparison are of the same type, the user can fix it by adding explicit casts. For instance in our running example, the right postcondition is one where the type coercion is made explicit:

```
Contract.Ensures(balance == (float)Contract.OldValue(balance) + amount);
```

From a programmer's point of view, this coercion seems redundant, as the expression `Contract.OldValue(balance)` + `amount` already has nominal type `float32` . He/she may expect that the explicit cast to `float32` would be a no-operation. But for the reasons explained in this paper, the cast may be a truncation.

4 Implementation and Experiment

We have implemeted the analysis described in this paper in Clousot, our static analyzer for CodeContracts [2]. The analyzer first reads the IL from disk, then constructs for each method the control flow graph, inserting contracts at the necessary points. Then it simplifies the program by getting rid of the evaluation stack and the heap, reconstructs expressions lost during compilation, and finally produces a *scalar* program.

Several analyses are run on the top of the scalar program, *i.e.* non-null, numerical, array, or arithmetic. We added the detection of precision mismatches in floating point comparisons described in this paper to the arithmetic analysis. The analysis is fast and accurate: on the core library of the .NET framework, `mscorlib.dll`, consisting of 25089 methods, it adds less than 10 seconds to the total time, and it reports 5 warnings. We manually inspected those, and they all represent real warnings similar to the following example:

```
bool IsNonZero(float f) {
  return f != 0.0F;
}
```

According to the ECMA standard, the parameter `f` of callers may be passed in registers and thus use more bits than `float32` in memory. Therefore, the inequality comparison against `0.0F` may result in `true`, even though `f` truncated to float32 is equal to 0.0F. As an example of where this could result in problems, consider:

```
class C {
  float a; // should never be 0.0F

  void Update(float f) {
    if (IsNonZero(f)) {
      this.a = f;
    }

  ...
```

In the above code, the programmer expects to guard the assignment to the field to make sure the value stored in the field is never 0.0F. However, due to register allocation of f, a value represented using more than 32 bits, close to 0.0F but not equal to 0.0F can pass the test and be truncated to 0.0F when stored into this.a.

5 Discussion

Some languages and compilers have addressed the problem described in this paper using other means. C compilers typically offer compile-time switches to control whether the compiler should emit code that adheres strictly to the bit-widths declared in types, or whether it is allowed to use extra precision for computations in registers.

Using strict adherence to the declared types requires compilers to emit truncating casts after each floating point operation that could result in more precision, or putting the floating point unit into a mode that automatically performs truncation [10]. The Java standard provides the strictfp keyword to enforce this truncating behavior and thereby avoids the problem described in this paper entirely at the cost of precision and speed.

We decided to detect problems with precision mismatches only when comparing floating point numbers. Another view would be to write an analysis that warns users whenever some higher precision float is implicitly cast to the nominal type width (effectively whenever a result is stored into memory). The problem with such an alternative analysis is that it would a) warn about most memory writes, b) miss implicit narrowings arising due to register spilling into memory introduced by the compiler. We found that warning about comparisons addresses the problem in a more actionable way.

6 Related Work

Previous work in this area focuses on enhancing static analysis techniques to soundly analyze programs with floating points. For instance [4,6,5] present static analyses to spot the source of imprecision in floating point computations. [9] introduces ideas to extend numerical abstract domains to the analysis of IEEE 754-compliant floating point values and [7] introduces program transformation techniques to reduce the error in floating point computations.

Our work is orthogonal to those in that we are not interested in the actual *values* of the computation but only in detecting situations that may cause unexpected comparison results. The need for such detection is particularly important to avoid runtime failures of contracts that seemingly have been validated via symbolic execution.

7 Conclusions

We described a simple static analysis to detect the absence of "surprising" behaviors of comparisons involving floating point numbers. The analysis is motivated by feeedback from users of our Code Contract tools. They reported false negatives similar to the example described in the introduction. We integrated the analysis in the static checker tool for Code Contracts, and experience shows it to be accurate enough.

Thanks. Many thanks to Matthias Jauernig, who was the first to report the unsoundess in Clousot illustrated here. Thanks also to Juan Chen for the detailed explanation of the floating point subsystem in the ARM architecture.

References

[1] ECMA. Common Language Runtime specification.

[2] M. Fähndrich and F. Logozzo. Static contract checking with abstract interpretation. In *FoVeOOS'10*, LNCS. Springer-Verlag, 2010.

[3] American National Standard for Programming Languages. The C language specification, 1990.

[4] E. Goubault and S. Putot. Static analysis of finite precision computations. In *VMCAI*, pages 232–247, 2011.

[5] M. Martel. Propagation of roundoff errors in finite precision computations: A semantics approach. In *ESOP*, pages 194–208, 2002.

[6] M. Martel. Static analysis of the numerical stability of loops. In *SAS*, pages 133–150, 2002.

[7] M. Martel. Program transformation for numerical precision. In *PEPM*, pages 101–110, 2009.

[8] B. Meyer. *Eiffel: The Language.* Prentice Hall, 1991.

[9] A. Miné. Relational abstract domains for the detection of floating-point run-time errors. In *ESOP'04*.

[10] D. Monniaux. The pitfalls of verifying floating-point computations. *ACM Trans. Program. Lang. Syst.*, 30(3), 2008.