The Threshold Algorithm: from Middleware Systems to the Relational Engine

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Abstract

The answer to a top-k query is an *ordered* set of tuples, where the ordering is based on how closely each tuple matches the query. In the context of middleware systems, new algorithms to answer top-k queries have been recently proposed. Among these, the Threshold Algorithm, or *TA*, is the most well known instance due to its simplicity and memory requirements. *TA* is based on an early-termination condition and can evaluate top-k queries without examining all the tuples. This top-k query model is prevalent over middleware systems, but also over plain relational data. In this work we analyze the challenges that must be addressed to adapt *TA* to a relational database system. We show that, depending on the available indexes, many alternative *TA* strategies can be used to answer a given query. Choosing the best alternative requires a cost model that can be seamlessly integrated with that of current optimizers. In this work we address these challenges and conduct an extensive experimental evaluation of the resulting techniques by characterizing which scenarios can take advantage of *TA*-like algorithms to answer top-k queries in relational database systems.

Index Terms

H.2.4 [Database Management]: Systems—Query processing; Relational databases; H.3.3 [Information Storage and Retrieval]: Information Search and Retrieval—Retrieval models; Search process.

I. INTRODUCTION

The answer to a *top-k query* is an *ordered* set of tuples, where the ordering is based on how closely tuples match the given query. Thus, the answer to a top-k query does not include all tuples that "match" the query, but instead only the best k such tuples. Consider for instance a multimedia system with an attribute R_C that contains information about the color histogram of each picture, and another attribute R_S with information about shapes. To obtain a few pictures that feature red circles, we might ask a top-10 query with a scoring function that combines (e.g., using a weighted average) the redness score of each picture (provided by R_C), with its "circle" score (provided by R_S). Other applications that rely on this model are information retrieval systems [1], data broadcast scheduling [2], and restaurant selection systems [3].

One way to evaluate a top-k query is to process all candidate tuples in the database, calculate their scores, and return the k tuples that score the highest (we called this a *scan-based* approach since it requires to sequentially examine all tuples in the database). In some scenarios, however, we can evaluate top-k queries more efficiently.

Reference [4] introduced the Threshold Algorithm, or *TA*, for processing top-k queries in the context of *middleware systems*. A middleware system in [4] consists of m autonomous data sources that provide access to attributes $\{c_1, \ldots, c_m\}$ of all objects in the system. Such sources can be explored using *sorted access* (in which attribute values are returned in descending order for a given attribute scoring function) and *random access* (in which the value of an object's attribute is obtained by providing the object's identifier). A top-k query in such scenario is given by a monotonic function F (e.g., min or average) that aggregates the individual attribute scores.

In a middleware system, we cannot efficiently combine the information about an object that is scattered across data sources. To *reconstruct* an object from its identifier, we need to issue several random accesses to the data sources (typically one random access per attribute), which is expensive. For that reason, scan-based algorithms are not an efficient alternative in middleware systems. The distinguishing characteristic of *TA* is the use of an early-termination condition that allows returning the top-k elements *without* examining the values of all tuples in the system. At the same time, *TA* requires a bounded amount of memory to operate (independent of the data size and distribution) and is straightforward to implement[1]. *TA* was shown to be *instance optimal* among all algorithms that use random and sorted access capabilities [4].

In recent years, there has been an increasing amount of research (e.g., [5], [6], [7], [8]) that focused on answering top-k queries in the context of relational database management systems, or RDBMS (we explain these previous efforts in the next section). These recent advances pose the intriguing question of whether *TA*, which was designed specifically for middleware systems, is suitable for a RDBMS. On one hand, the early termination condition, memory guarantees, and overall simplicity of *TA* are very appealing. On the other hand, the *reconstruction problem* of scan-based algorithms is not that severe in a RDBMS. In fact, we can centralize all the information about an object in a table or materialized view so that each row contains all attributes of the object being represented. Therefore, we do not need to issue expensive random access probes to retrieve missing attribute values as it is the case in middleware systems. In this work we critically examine when *TA*-like algorithms are a good alternative in a RDBMS, and when they fail to provide any benefit.

[1]Alternative algorithms, such as NRA or CA [4], require in the worst case as much memory as the data size itself, and rely on complex bookkeeping which make implementations non-trivial.

In Section II we first define the top-k query problem. We then determine, in Section III, how the different components of *TA* reconcile with algorithms and data structures present in a RDBMS, and what are the main differences between the middleware solution and its relational counterpart. As we will see, depending on the available indexes, many alternative TA-*strategies* are possible. Not only the number of such strategies is typically very large, but the cost of each alternative can vary drastically. (*TA* strategies can be either orders of magnitude more efficient or more expensive than scan-based alternatives.) It is then crucial to develop a cost model that helps the query optimizer decide among all *TA* strategies and the traditional scan-based alternative. In the second half of Section III we develop a cost model that uses power-laws to approximate the cost of a *TA* strategy in a way that it is easy to incorporate to an existing optimizer. Finally, In Section IV we discuss some extensions to the basic model and we report extensive experimental results in Section V.

A. Related Work

There has been a significant amount of research on top-k queries in the recent literature. Although in this work we refer to the middleware algorithm as *TA* [4], we note that slight variations of *TA* were independently proposed in [9] and [10]. Later, references [11] and [3], among others, extended the basic algorithms to address middleware sources that restrict their access capabilities. Reference [12] conducted an experimental evaluation of an earlier algorithm [13] that efficiently answers top-k queries. The authors implemented the algorithm in the Garlic middleware system and explored its strengths and weaknesses. Later, reference [4] showed that *TA* is asymptotically more efficient than the algorithm in [13].

Reference [6] presented techniques to evaluate top-k queries through traditional SQL `order by` clauses. These techniques leverage the fact that when k is relatively small compared to the size of the relation, specialized sorting (or indexing) techniques that can produce the first few values efficiently should be used. However, in order to apply these techniques for the types of scoring functions that we consider in this work, we need to first evaluate the scoring function for each database object. Hence, these strategies require a sequential scan of all data as a preprocessing step.

References [7], [8] proposed a different approach to evaluate top-k selection queries in database systems: a thin layer on top of the database system maps top-k selection queries into tight

range queries that are expected to contain the top-k answers. While [7] uses multidimensional histograms to do the mapping, reference [8] relies on samples. If some of the top-k results are missed by the approximated range query, [7] restarts the top-k query with a larger range (or a full sequential scan is performed). In contrast to [7], the techniques presented in this work do not result in restarts and therefore avoid sequential scans.

Reference [5] focused on a problem closely related to ours, which is integrating top-k join queries in relational databases. The assumption in that work is that the contribution of each table to the overall scoring function depends on just one attribute in the table (or, at least, that we can scan each table in descending order of the "partial score" associated with the table). Therefore, the main problem consists of introducing non-blocking rank-aware join operators and reorder the joins in the final execution plan to minimize the overall cost. In contrast, we focus on multi-attribute top-k queries over a single table, and investigate how we can exploit indexes to evaluate such queries efficiently. As a motivating example, consider an art museum that displays both pictures and photographs. We might want to locate a room in the museum that contains both a picture with red circles and a photograph with blue squares (let us define the combined score of a room as the average score of its best "red-circle" picture and "blue-square" photograph). We might then issue the following query:

```
SELECT TOP 1 PI.room_location
FROM pictures PI, photographs PH
WHERE PI.room_id = PH.room_id
ORDER BY 0.5*( red_circle(PI) + blue_square(PH) )
```

The techniques in [5] would efficiently answer this query provided that there is a mechanism to retrieve pictures in descending order of *"red_circle"* values (similarly, photographs in descending order of *"blue_square"* values). As we saw in the introduction, the "red_circle" score of a picture is not given by any of its attributes in isolation, but instead by another scoring function on multiple attributes. Therefore, if we were to apply the techniques in [5], we would need to first scan both `pictures` and `photographs` tables, then sort all the objects by their respective partial scoring functions, store these partial scores as computed columns, and finally pipeline this intermediate result to the rank-aware join operator of [5]. Of course, scanning and sorting the leaf nodes in the plan can easily become the bottleneck. In this paper we show how to efficiently process such sub-queries. We believe that our work and [5] complement each other nicely: the

resulting execution plans returned by our techniques can be seen as the leaf nodes in the join plans of [5], thus extending the set of top-k queries that can be evaluated without scanning the underlying tables.

Recently, reference [14] proposed to make *TA* cost-aware by explicitly modeling the cost of sorted accesses and allowing *TA* to vary the relative access *speed* to the sorted sources accordingly. For instance, if a source suddenly becomes very slow, the algorithm in [14] would reduce the frequency that this source is accessed, therefore increasing the efficiency of the whole algorithm. Reference [14] has some similarities with our work, but it is still focused on middleware scenarios and therefore does not take into account the specific characteristics of relational database systems. First, reference [14] assumes that sorted access costs fluctuate widely and are in general much larger than random access costs. In contrast, in a RDBMS the cost of (random or sequential) accesses are very predictable and do not fluctuate significantly (additionally, the cost of a random access is known to be much higher than that of a sequential access in a RDBMS). Second, reference [14] adapts the algorithm on the fly, while the query is executing. In contrast, our techniques exploit global statistics that are present in a RDBMS to statically choose the best execution plan. Finally, in contrast with a middleware scenario, our work recognizes that there might be different ways to accomplish a random or sorted access depending on the set of available indexes in the RDBMS, and this choice can greatly impact the overall efficiency of the resulting algorithm. We believe, however, that our work can benefit from the ideas in [14]. In fact, after the best plan has been picked, we could use the techniques in [14] to vary the relative speed at which the chosen indexes are accessed and further improve the efficiency of the algorithm (see Section IV-B for more details).

II. Problem Statement

We next review the *TA* algorithm for middleware systems and formalize our problem in the context of a RDBMS.

A. TA Algorithm

Consider a top-k query over attributes $c_1, \ldots, c_m$ and a scoring function $F(s_1(c_1), \ldots, s_m(c_m))$, where s_i is the scoring function of attribute c_i. Suppose that source R_i handles attribute c_i. Below we summarize the *TA* algorithm to obtain the k objects with the best scores.

1) Do sorted access in parallel to each of the m sources. As an object O is seen under sorted access in source R_i, do random access to the other sources R_j and calculate the remaining attribute scores s_j of object O. Compute the overall score $F(s_1, \ldots, s_m)$ of object O. If this value is one of the k highest seen so far, remember object O and its score (break ties arbitrarily).
2) Let s_i^L be the score of the last object seen under sorted access for source R_i. Define the threshold value T to be $F(s_1^L, \ldots, s_m^L)$. If the score of the current top-k object is worse than T, return to step 1.
3) Return the current top-k objects as the query answer.

Correctness of *TA* follows from the fact that the threshold value T represents the best possible score that any object not yet seen can have, and *TA* stops when it can guarantee that no unseen object might have a better score than the current top-k ones. The algorithm in [4] was later extended to support scenarios in which some sources cannot provide sorted access. In such situations, we need to replace s_i^L with the maximum possible score for each source R_i that cannot provide sorted access. The correctness and instance optimality of *TA* are preserved in this case as well.

B. Server Side Problem Statement

In the context of a RDBMS, we focus on the following problem. Consider table R with attributes $c_1, \ldots, c_m$ (and possibly other attributes not mentioned in the query). We specify a top-k query in SQL as follows[2]:

```
SELECT TOP(k) c1, ..., cm
FROM R
WHERE P
ORDER BY F(s1(c1), ..., sm(cm)) DESC
```

where P is an filter predicate, F is a monotonic aggregate function, and $\mathtt{s}_i$ are individual attribute scoring functions. In current database systems, the best way to evaluate such top-k query is a scan-based approach. Conceptually, we first obtain the best execution plan for the following query:

[2]Queries might have additional columns in the SELECT clause that are not present in the ORDER BY clause. This results in minor variations to our techniques, but we omit such details for simplicity.

```
SELECT c1,..., cm, F(s1(c1),..., sm(cm)) as S
FROM R
WHERE P
```

and then we apply a *partial-sort*[3] operator on column S on top of this execution plan. If no predicate P is specified, the best execution plan consists of a sequential scan over table R, or over the smallest-sized index that covers all columns of interest if any is available (since columns have different widths, the smallest-sized index need not agree with the one with the fewest number of columns). In the next section we focus on queries with no filter predicate P, and in Section IV we explain how to lift this restriction.

Analogous to the middleware scenario, execution alternatives adapted from *TA* might sometimes outperform the scan-based approach used in current database systems. We next investigate how to adapt *TA* to work in a RDBMS, what are the differences and similarities with respect to the middleware case, and when *TA*-based strategy can be beneficial in this new scenario.

III. TA IN A RDBMS

We now explain how *TA* can be adapted to work in the context of a RDBMS. Indexes are the most widely used mechanism to improve performance in a RDBMS, and our adaptation of the *TA* algorithm is based on such physical structures. Specifically, we *simulate* each autonomous source in the original *TA* with suitable index strategies that support the sorted and random access interfaces.

Recall that *TA*'s scoring function is a monotonic aggregate of individual attribute scoring functions. If we do not restrict these attribute scoring functions in any way, in general we would need to access all tuple values and sort them by each attribute scoring function before providing the first sorted access. This preprocessing step is expensive and defeats the purpose of *TA* (i.e., avoiding costly scans over the data)[4]. For this reason, we restrict the attribute scoring functions to those that can be evaluated efficiently by an index structure (specifically, each sorted access should be processed in a constant amount of time).

[3]A *partial-sort* operator scans its input and uses a priority queue to maintain the k tuples with the largest values in a given column.

[4]In middleware systems, cost metrics do not include this preprocessing step since it is assumed to be done by the autonomous sources.

Consider for instance an attribute scoring function $3 \cdot c_1$. If an index with leading column c_1 is available (i.e., an index whose primary order is by c_1), we scan this index in descending order to obtain each tuple sorted by $3 \cdot c_1$ in constant time. As a slightly more complex example, if the scoring function is $-(c_1 - 0.5)^2$ and an index with leading column c_1 is available, we proceed as follows[5]. Initially, we seek the index for value 0.5 and return all the matches. Then, we maintain two pointers that traverse the index in opposite directions starting at c_1=0.5, and we return, for each subsequent sorted access, the current tuple from one of the two traversals that is closer to c_1=0.5 (see Figure 1). Although in this work we focus on simple functions, the methodology is the same for complex functions as long as they are efficiently supported by indexes.

After obtaining a new tuple from each source, *TA* performs random accesses to the other sources to obtain the remaining attribute scores. There are two differences in this step between the original *TA* algorithm and our adapted version. First, in a RDBMS we can obtain all remaining attribute values at once using a single (random) primary index lookup rather than $m - 1$ random accesses. Second, if the index providing sorted access is a covering index (i.e., it is defined over all c_i columns), we do not even need a primary index lookup to obtain the remaining values.

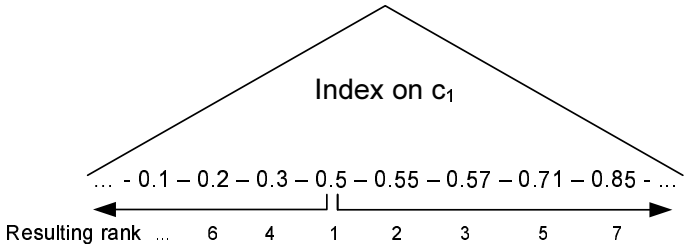


Fig. 1. Providing sorted accesses using indexes.

After defining how to provide sorted and random access by using indexes, we need a piece of logic that puts everything together. We define a new physical operator, which we call RTA (for Relational-TA), illustrated in Figure 2. The RTA operator, analogous to the original *TA* algorithm, (i) retrieves a new tuple from each input in a round robin fashion, (ii) maintains a priority queue with the top-k tuples seen so far, (iii) calculates the threshold value, and (iv) stops when the threshold value is not better than the current top-k value. Figure 2 shows a sample

[5]We assume that larger scores are better, so we introduce a negative sign in $-(c_1 - 0.5)^2$ to first obtain tuples closer to c_1=0.5. In general, the best scores could either be the largest or smallest ones, but we omit those details for simplicity.

execution plan for a top-k query with scoring function $\sum_{i=1}^{3} -(c_i - 0.5)^2$. In the example, the index strategy for column c_1 does not use a covering index, and therefore primary index lookups are necessary. Because there is no index strategy for attribute c_3, RTA uses in the threshold formula the maximum possible score for c_3, which in this case is zero.

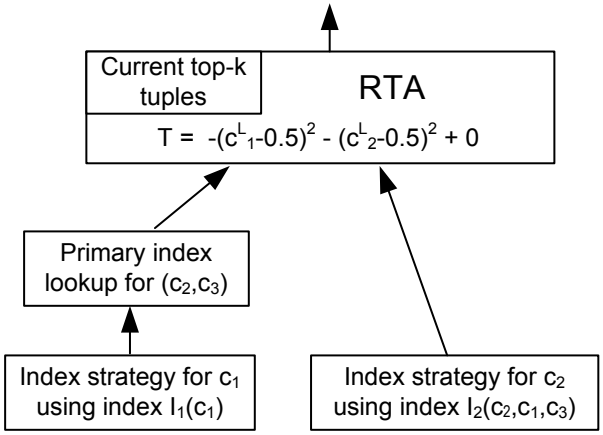


Fig. 2. A sample *TA*-based execution plan.

Any *TA* strategy that uses zero or one index with leading column c_i for each attribute c_i in the scoring function is a candidate execution plan. In the next section we define the space of candidate *TA* strategies with respect to the set of available indexes and describe some pruning techniques that reduce the number of alternatives to consider.

A. Space of TA Strategies

Suppose that we associate, with each column c_i in the scoring function, the set of indexes that have c_i as their leading column. We can then obtain a valid *TA* strategy by picking zero or one index from each group. If n_i is the number of indexes with leading column c_i, the total number of plans is $\Pi_i(n_i + 1) - 1$. We now introduce two simple properties that allow us to reduce the set of candidate indexes.

Property 1: If there is a covering index I with leading column c_i (or there are many covering indexes but I is the smallest-sized of those) and some *TA* strategy P uses another index I' for column c_i, we can replace I' with I to obtain a more efficient strategy.

In fact, a covering index provides all the necessary columns and therefore does not require primary index lookups (see Figure 2). Since each entry in the covering index is smaller than the

corresponding one in the primary index, the number of pages accessed using the covering index is no larger than the number of pages accessed by a non-covering index and the primary index lookups put together. Similarly, if many covering indexes are available, the smallest-sized one accesses the least number of pages. Therefore, the strategy that uses the I is more efficient than any of the alternatives.

Property 2: Suppose that there is no covering index with leading column c_i. If I is the smallest-sized index with leading column c_i and some *TA* strategy P uses another index I' for column c_i, we can replace I' with I to obtain a more efficient strategy.

In fact, if no covering index is available for c_i, then *any* index that handles attribute c_i would miss at least one attribute, and therefore would require one (and only one) primary index lookup to obtain the remaining attribute scores. The smallest-sized index with leading column c_i guarantees that the index traversal is as efficient as possible. In addition, independently of the index used for c_i, the subsequent operators (including the primary index lookups) are the same. Therefore, the smallest-sized index with leading column c_i results in the most efficient execution.

Using both properties, the space of candidate *TA* strategies is reduced, for each attribute c_i in the scoring function, to either zero or the best index for c_i. We next show that in absence of additional information, every possible plan is optimal for some data distribution. Consider the following table and a scoring function $F = \sum_{i=1}^{m} c_i$.

id	c_1	c_2	c_3	...	c_n	c_{n+1}	...	c_m
1	1	0	0	...	0	0	...	0
2	0	1	0	...	0	0	...	0
3	0	0	1	...	0	0	...	0
...	...	...	...	...	...	...	...	...
n	0	0	0	...	1	0	...	0
n+1	0.9	0.9	0.9	...	0.9	0	...	0
n+2	0.9	0.9	0.9	...	0.9	0	...	0
...	...	...	...	...	...	...	...	...

Assuming that the table is of size N, the top-k tuples (for $k < N$-n) will have score $0.9n$. Suppose that we use indexes for columns c_1 through c_n. In this case, after $k + n$ iterations the top-k values will have score $0.9n$ and the threshold would also be $0.9n$ (because the maximum

score for all columns without indexes is zero). Therefore, after $k + n$ iterations the algorithm would terminate. Now, if we use indexes over any subset of $\{c_1, c_2, \ldots, c_n\}$, the threshold value after any number of iterations beyond n would be $0.9n_1 + 1(n - n_1)$ for some $n_1 < n$, which is larger than the current top-k value $0.9n$. Therefore, this strategy would require reading all N tuples from such indexes and would in general be more expensive than the previous one. Finally, we note that any plan that uses indexes from c_{n+1} through c_m will be more expensive than if those indexes are not used, because they do not reduce the threshold and therefore the number of iterations remains the same while the number of index sorted accesses increase. In conclusion, the best possible *TA* strategy uses indexes just for c_1 through c_n. But those columns are arbitrary, so for any candidate *TA* strategy there will be a data distribution for which it is optimal.

More importantly, depending on the data distribution and available indexes, the best possible *TA* strategy might be worse than the scan-based alternative. In fact, consider the table above, and suppose that $N = n + k$. In this case, the strategy that uses all indexes c_1 through c_n is still optimal, but it terminates after $n + k = N$ iterations (in fact, any *TA* strategy terminates after N iterations for this specific data set). Depending on the available indexes, each iteration may require a number of random accesses. Therefore, the optimal strategy would require at least as many random accesses as objects in the original table. The best *TA* strategy in this case will therefore take more time than a simple *Scan* alternative. For that reason, it is crucial that we develop a cost model that not only allows us to compare candidate *TA* strategies, but also help us decide whether the best *TA* strategy is expected to be more efficient than the scan-based alternative. In the next section we introduce such cost model.

B. Cost Model

Although *TA* strategies are new to an RDBMS, they are composed of smaller pieces that are well-known and implemented in current systems. Any execution of *TA* consists of a series of sorted accesses (i.e., index traversals), a series of random accesses (i.e., optional primary index lookups in absence of covering indexes), and some computation to obtain scores, threshold values, and priority queue maintenance (i.e., scalar computation and portions of the *partial-sort* operator). If a given *TA* strategy requires D iterations to finish (i.e., D sorted accesses over each

attribute), the cost of its execution plan is estimated as follows:

$$Cost = D \cdot T_C \cdot \left(\sum_{\substack{1 \leq i \leq m, \\ c_i \text{uses index} \\ \text{strategy}}} T_{S_i} + \sum_{\substack{1 \leq i \leq m, \\ c_i \text{uses non} \\ \text{covering index}}} T_L \right)$$

where T_C represents the cost of maintaining the priority queue and calculating scores and threshold values, T_{S_i} is the cost of a sorted access using the index for attribute c_i, and T_L is the cost of a primary index lookup. We now explain how to exploit this formula to further reduce the candidate plans that we need to consider during optimization.

Consider two *TA* strategies T_1 and T_2, and suppose that the set of attributes that T_1 handles using indexes is a subset of that of T_2. Let D_1 and D_2 be the number of iterations required by T_1 and T_2, respectively, to obtain the top-k tuples. The cost of T_1 is then estimated as $D_1 \cdot \gamma_1$ where D_1 is the number of iterations that T_1 requires and γ_1 represents the cost per iteration (see the cost formula above). Similarly, the cost of T_2 is estimated as $D_2 \cdot \gamma_2$. A property derived from the *TA* algorithm is that $D_1 \geq D_2$. The reason is that T_1 has less information to calculate threshold values, and therefore requires more iterations to make the threshold fall below the current top-k score. At the same time, $\gamma_1 \leq \gamma_2$ because, at each iteration, T_1 makes sorted and possibly random accesses to a subset of the indexes present in T_2. Thus, in general, either T_1 or T_2 can be the better alternative. Now consider strategy T_3, whose set of indexes includes that of T_1 and it is included in that of T_2. Using similar arguments as above, we can guarantee that the cost of T_3 would be at least $D_2 \cdot \gamma_1$. Therefore, if the best strategy found so far is cheaper than $D_2 \cdot \gamma_1$, we can omit from consideration all intermediate strategies like T_3.

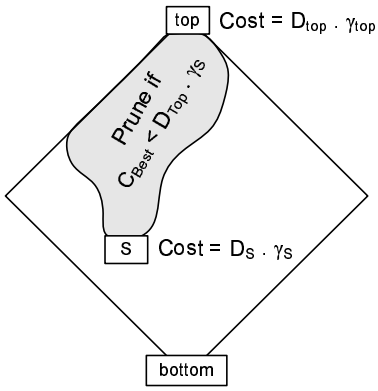


Fig. 3. Pruning *TA* strategies.

In practice, we can use this property as follows. Figure 3 shows a lattice on the subset of indexes used by *TA* strategies. Suppose that we initially evaluate the cost of strategy *Top*, which uses indexes for all columns, and obtain D_{Top} as the estimated number of iterations. Assume that the best strategy found so far has cost equal to C_{best}. Whenever we consider a new strategy S, we can calculate the cost per iteration γ_S. If $D_{Top} \cdot \gamma_S > C_{best}$ we can safely omit from consideration all *TA* strategies that include S's indexes.

In the above cost formula, all components can be accurately estimated reusing the cost model of the database optimizer, except for the number of iterations D. This value depends on the data distribution and crucially determines the overall cost of *TA*. We now show how to estimate the expected number of iterations D for a given *TA* strategy.

C. Estimating the Complexity of TA

As explained above, in order to estimate the cost of any given *TA* strategy, we need to approximate the number of iterations D that such strategy would execute before the threshold value falls below the current (and therefore final) top-k score. In other words, if we denote s_k the score of the top-k tuple and $T(d)$ the value of the threshold after d iterations, we need to estimate D, the minimum value of d such that $T(d) \leq s_k$. We will find D by approximating the two sides of this inequality. We first approximate s_k, the score of the top-k tuple. Then, we estimate the minimum value d after which $T(d)$ is expected to fall below s_k.

Let us first assume that we already estimated s_k, the score of the top-k tuple. To approximate the number of iterations after which the threshold value falls below s_k we use single-column histograms. In fact, an important observation regarding *TA* is that it manipulates single-dimensional streams of data returned by the autonomous sources, and therefore exploiting single-column histograms for the approximation does not implicitly assume independence. Thus, we can accurately simulate *TA* over histograms very efficiently, at bucket granularity. Figure 4 shows a simple example for a scoring function $F(c_1, c_2) = c_1 + 5 \cdot c_2$. If we are told that the top-k score s_k is equal to, say, 0.95, we can walk the histogram buckets starting from the right-most bucket and obtain the minimum number D for which the threshold calculated with the values of the top-D tuples from each histogram is smaller than s_k. For simple attribute scoring functions, this procedure requires splitting buckets and using interpolation in a similar way as when joining histograms for cardinality estimation (the complexity is on the order of the number

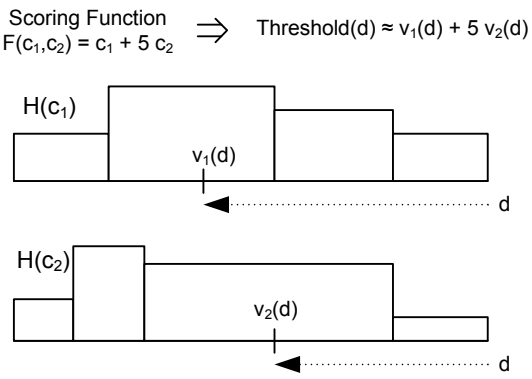


Fig. 4. Approximating D using histograms.

of histogram buckets). For complex attribute scoring functions, we use binary search over the histogram domains to find D, which is also very efficient.

As we will see, estimating the score s_k of the top-k tuple is more complex. The difficulty resides on the fact that this is a truly multidimensional problem, and therefore relying on single-dimensional approximations (like traditional histograms) can easily result in excessive approximation errors. For that reason, we need to use multidimensional data synopses to estimate s_k values. We considered multidimensional histograms and samples as alternative synopses, and experimentally determined that samples provided the best results. The reason is that multidimensional samples scale better with increasing number of dimensions. In fact, it is known that multidimensional histograms do not provide accurate estimations beyond five to six attributes (e.g., see [15], [16]) and therefore we would need to build a large number of at most 5-dimensional histograms to cover all attribute combinations in wide tables. We note that, unlike previous work in approximate query processing, our precomputed samples must be very small. The reason is that these samples are loaded and manipulated as part of query optimization, and therefore must be roughly of the size of other database statistics. In other words, single-column projection of the precomputed sample must be roughly as large as a single-column histogram in the database system. In all our experiments, we use a sample size of 1,000 tuples, independently of the size of the underlying table.

To estimate the value s_k from the precomputed sample we proceed as follows. We first compute the score of each tuple in the sample and order the scores by decreasing score value. We then estimate s_k using interpolation. If the original table contains N tuples and the sample contains S tuples, the probability that the top-r_S element in the sample is the top-r_N element in the original

data, denoted $P(r_S \text{ is } r_N)$, is given by:

$$P(r_S \text{ is } r_N) = \frac{\binom{r_N-1}{r_S-1} \cdot \binom{N-r_N}{S-r_S}}{\binom{N}{S}}$$

and therefore, the expected rank in the original table of the top-r_S tuple in the sample, denoted $E(r_S)$, is given by:

$$E(r_S) = \sum_i \frac{\binom{i-1}{r_S-1} \cdot \binom{N-i}{S-r_S}}{\binom{N}{S}} \cdot i = r_S \cdot \frac{N+1}{S+1}$$

Using this equation, we locate the two consecutive samples in descending order of score that are expected to cover the top-k element in the original data and interpolate their respective scores to obtain an approximation of s_k. The estimation algorithm can then be summarized as follows:

1) Using a precomputed small sample, obtain the approximate score s_k of the top-k tuple.
2) Using single-column histograms, obtain the minimum value D that results in a threshold value below s_k.
3) Evaluate the cost function using D as the approximation of the number of iterations.

We note that this algorithm can be used for arbitrary *TA* strategies, by using in step 2 only the histograms that correspond to columns that are handled using indexes in the query plan (the remaining columns simulate *TA* by using the maximum possible score). On the other hand, the above algorithm presents some problems for the class of top-k queries that we address in this work. In the next section we explain these obstacles and how we address them.

*D. Addressing Small **k** Values*

Unfortunately, the algorithm of the previous section has an important drawback in most interesting query instances. The problem is that only around a thousand tuples are used in the precomputed sample, and therefore we are always working on the tails of probability distributions. For instance, suppose that our data set contains 1 million tuples, and we use a sample of size 1,000. In this case, the top scoring tuple in the sample is expected to rank at position $\frac{1{,}000{,}001}{1{,}001} \approx 999$ in the original table. This means that for any top-k query with $k < 999$ (i.e., a very common scenario), s_k would likely be smaller than the top tuple in the sample. In this situation, we would need extrapolate the top-k score. As we show in the experimental section, this procedure generally results in large approximation errors. In this section we discuss two complementary approaches to address this issue.

Interpolating Iterations rather than Object Scores

We next propose an alternative approximation mechanism that works in the space of iterations of *TA* rather than on the space of object scores. We first obtain the top-k' scores from the precomputed sample where k' is a small constant number (we use $k' = 10$ in our experiments). If the original table contains N tuples and the sample size is S, the top-k' scores in the sample are expected to rank in the original table at positions $i \cdot \frac{N+1}{S+1}$ for $i = 1..10$. We then calculate as before the approximated number of iterations for each such score and obtain a set of pairs $\{(i \cdot \frac{N+1}{S+1}, \text{expected iterations for } i \cdot \frac{N+1}{S+1}), i = 1..10\}$. Finally, we fit the best curve that assigns expected number of iterations to obtain top-$\mathcal{K}$ values for arbitrary values of $\mathcal{K}$, and evaluate this function at $\mathcal{K} = k$. We use the obtained value as the estimated number of iterations to obtain the top-k elements. The revised algorithm can be summarized as follows:

1) Using a precomputed small sample, obtain the top-10 scores that are expected to rank in the original data at positions $\frac{N+1}{S+1}, \ldots, \frac{10 \cdot (N+1)}{S+1}$.
2) Using single-column histograms, obtain the expected number of iterations for each score.
3) Find the best curve that approximate the "number of iterations" function and evaluate it in k obtaining D.
4) Evaluate the cost function using D as an approximation for the number of iterations.

An important decision is which model function to use to approximate number of iterations for arbitrary values of k. The simplest approach is to use linear regression, but many others are possible. After an extensive experimental evaluation, we decided to use a power function $D(k) = a \cdot e^{b \cdot k}$ where a and b are parameters that minimize the square root error[6]. Of course, this is just an approximation as the true underlying function depends on the data distribution and in general can have any (monotonically increasing) behavior. However, we found that power laws resulted in the best overall accuracy for a wide class of data distributions and scoring functions (see Section V for some examples that validate our choice of model functions).

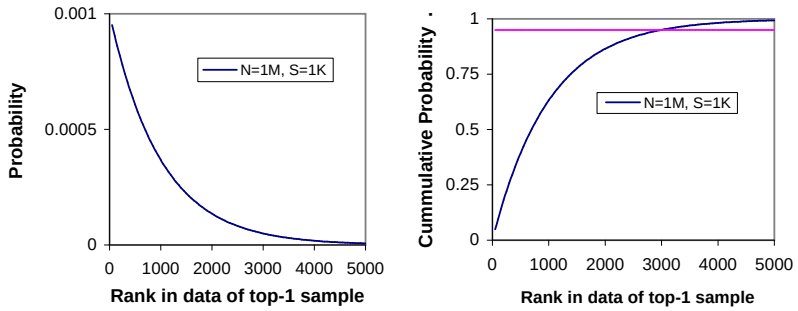
[6]Note that [13] introduces an earlier algorithm to obtain top-k answers with a probabilistic access cost that follows a power law in terms of k.

Using a Safety Factor for Large Rank Variance

A second difficulty that our techniques face arises from the limited sample size. As we explained earlier, the expected rank in the data distribution of the i-th highest ranked element in the sample is $i \cdot \frac{N+1}{S+1}$. We now briefly analyze how accurate that estimation is for the special case of i=1 (i.e., how close to $\frac{N+1}{S+1}$ in the original data set is the rank of the top object in the sample). The variance of the rank in the original table of the top tuple in the sample, denoted $V(1_S)$, is given by:

$$V(1_S) = \sum_i \frac{\binom{N-i}{S-1}}{\binom{N}{S}} \cdot \left(\frac{N+1}{S+1} - i\right)^2 = \frac{S(N+1)(N^2 - NS - 3N + S^2 + 2)}{(S+1)^2(S+2)(N-S+1)}$$

which is approximately $\left(\frac{N+1}{S+1}\right)^2$ if $N \gg S$. In other words, the standard deviation of the rank in the original relation of the top object in the sample is as large as the expected rank itself. Figure 5 illustrates this behavior for a hypothetical table with 1 million tuples and a sample of size 1,000. In this case, the rank in the original table of the highest ranked sample is expected to be around 999. Figure 5(a) shows the probability distribution of such rank. We can see that 999 is not even the most frequent value, and that the probability distribution is rather flat. Figure 5(b) shows the cumulative probability distribution and the fact that with 95% confidence the actual rank of the top ranked sample can be anywhere between 1 and 3000 (a range that is three times as large as the actual expected value).



(a) Probability. (b) Cumulative Probability.

Fig. 5. Rank estimation using samples.

Although this issue seems very problematic, the ultimate goal of our estimation procedure is not to get very accurate values for the number of iterations, but to help the optimizer choose a good execution plan. Therefore, if the actual estimated cost of the best *TA* strategy is significantly

larger or smaller than that of the scan-based alternative, moderate variations in the estimation of cost will not change the optimal plan from being chosen. When the two numbers are close, though, there is a larger chance that errors in estimation will propagate to the selection of plans. We examine these issues in the experimental section.

We use an additional small "safety" factor as follows. Recall that the procedure to estimate the cost of any *TA* strategy uses some approximations that are intrinsically not very accurate. On the other hand, the cost estimation for a scan alternative can be estimated very accurately. As a conservative measure, we only choose a *TA* strategy if it is cheaper than α times the cost of an alternative scan, where $0 < \alpha \leq 1$ (we use $\alpha = 0.5$ in our experiments, but this number can in general reflect the degree of confidence that we require before executing a *TA* strategy). We choose a *TA* strategy only if we are confident enough that it will be cheaper than the scan-based alternative. In doing so we might choose the scan-based alternative even though a *TA* strategy was indeed the optimal plan, but this is an expected consequence of our conservative approach. We analyze this tradeoff experimentally in Section V.

IV. EXTENSIONS

In this section we discuss some extensions that can be easily incorporated to our model.

A. Handling Filter Predicates

In the previous section we assumed that there was no filter predicate restricting the set of tuples that qualify to be part of the answer. We now describe how we can relax that restriction. Consider the general query:

```
SELECT TOP(k) c1, ..., cm
FROM R
WHERE P
ORDER BY F(s_1(c1), ..., s_m(cm)) DESC
```

where P is an arbitrary predicate over the columns of R. We now discuss how we can evaluate such queries and then how we can estimate their execution cost.

1) Execution Alternatives: A straightforward extension of *TA* to handle arbitrary filter predicates is as follows. Recall from Figure 2 the main components of a typical *TA* strategy. The idea is to add a small piece of logic in the RTA operator, which evaluates predicate P before

considering the addition of the current object to the set of top-k tuples. If the current tuple does not satisfy P, it is dropped from consideration. The calculation of the threshold, however, considers all tuples whether they satisfy P or not. This execution strategy is feasible if RTA receives input tuples that additionally reference all the columns required to evaluate P. If the current tuple is obtained from an index strategy followed by a primary index lookup, then all relevant columns are already present. Otherwise, if a covering index is used, we need to perform an additional index lookup to obtain the remaining columns. Of course, if the covering index additionally contains all columns referenced by P, then there is no need to do any primary index lookup. All the pruning considerations are similar to the original case discussed in Section III-B.

If the predicate P satisfies certain properties, there is a different alternative that might be more efficient, especially if P is selective. Consider the following query:

```
SELECT TOP(10) a, b, c
FROM R
WHERE d=10
ORDER BY a+b+c DESC
```

and suppose that an index on column d is available. In that case, we can push the selection condition into the order by clause and transform the query above as follows:

```
SELECT TOP(k) a, b, c
FROM R
ORDER BY a+b+c+{ 0    if d=10
               { −∞   otherwise   DESC
```

It is fairly simple to see that, if there are at least k tuples for which d=10 both queries are equivalent (otherwise, we need to discard from the latter query all the tuples with score equal to $-\infty$). In this case, we reduced a query with a filter predicate to an equivalent one that does not have it. Also, the index on column d can be used to return all tuples that satisfy the filter predicate before any tuple that does not satisfy it[7]. In general, we can use this alternative *TA* strategy by pushing to the scoring function all predicates that can be efficiently retrieved using indexes.

[7]After the first tuple that not satisfying P is returned, the threshold value drops below the current top-k score and execution terminates.

2) Cost Estimation: A key observation to estimate the cost of the extended strategies discussed above is that introducing filter predicates does not change *TA*'s early termination condition, which is "stop after the threshold falls below the score of the current (and final) top-k object". Therefore, the procedure explained in Section III-B can also be used in this scenario. The difference is that initially we need to apply predicate P to the precomputed sample so that we only consider tuples that satisfy the filter predicate. Note, however, that the accuracy of this method will diminish since the effective sample that we use for estimation is smaller than the original one. For instance, a very selective predicate might filter out all but five tuples in the original sample S. The actual number of tuples in the resulting sample S' would therefore be too small, which makes our techniques more difficult to apply (e.g., recall from Section III-D that we need at least ten elements in the sample to fit the best power law that determines the expected cost of a *TA* strategy).

B. Other Enhancements

There are other enhancements to the main algorithm that can speed up the overall execution of a top-k query. For instance, *TA* uses bounded buffers and only stores the current top-k objects in a priority queue. While this is important to guarantee that the algorithm will not run out of memory during its execution, some objects might be looked up in the primary index multiple times if they are not part of the top-k objects (once each time an object is retrieved from a source). We can trade space for time by keeping a hash table of all objects already seen. If we retrieve an object using sorted access and it is already in the hash table, we do not process it again (saving a primary index lookup).

An optimization studied in the literature is to relax the requirement that sorted accesses are done in a round-robin fashion [14]. In general, we can access each source at different rates, and retrieve tuples more often from those sources that contribute more to decreasing the threshold value. Conversely, if the score distribution of an attribute is close to uniform, we can decrease the rate at which we request new tuples from that source. It is fairly straightforward to show that a strategy that performs sorted accesses at different rates can be better than any alternative strategy that proceeds in lockstep. It is interesting to note that for fixed, but different access rates to the sources, the estimation technique of Section III-B that uses single-column histograms can be applied with almost no changes. In this case, we would obtain the minimum values

$\{D_1, \ldots, D_m\}$ for which the threshold value calculated with the top-D_i value for each attribute c_i is expected to fall below the top-k score.

Finally, there are opportunities to further avoid primary index lookups. Consider a top-k query over attributes a, b, and c, and suppose that we use a composite non-covering index on (a, b) to process attribute a. For each tuple retrieved from the index, we can first assume that the value of c is as large as possible. If the resulting score calculated in that way is still smaller than the current top-k score, we can discard the tuple without performing a primary index lookup. Although this idea is at first glance straightforward, it carries important implications for optimization. Specifically, our pruning techniques of Section III-A need to be refined to work in this scenario. For instance, if indexes over both (a) and (a, b) are available for a top-k query over attribute a, b, and c, our pruning techniques would not consider (a, b) (see Property 2), which might be the best alternative when this optimization is used.

Implementing these enhancements and providing suitable extensions to the cost model is part of future work.

V. Experimental Evaluation

We next report an extensive experimental evaluation of our techniques, which we implemented in Microsoft SQL Server. We note that, aside from our own extensions, we did not use vendor-specific features in our experiments. We therefore believe that our results would be directly applicable to other implementations of SQL that support multi-column indexes. Below we detail the data sets, techniques, and metrics used for the experiments of this section.

Data Sets: We use both synthetic and real data sets for the experiments. The real data set we consider is `CovType` [17], a 55-dimensional data set used for predicting forest cover types from cartographic variables. Specifically, we consider the following quantitative attributes for our experiments: elevation, aspect, slope, horizontal and vertical distances to hydrology, horizontal distances to roadways and fire points, and hill shades at different times of the day. The cardinality `CovType` is around 545,000 tuples.

To evaluate specific aspects of our techniques we also generated a synthetic data distribution with 1 million tuples and 14 attributes with different distributions and degrees of correlation. Table I described the attributes of the synthetic database. Multi-Gaussian distributions consist of a number of overlapping gaussian bells, where the number of tuples on each bell is regulated

Column	Description
`unif1, unif2`	Random(0, 1)
`corr_0.1`	`unif1` + Random$(-0.0005, 0.0005)$
`corr_1`	`unif1` + Random$(-0.005, 0.005)$
`corr_10`	`unif1` + Random$(-0.05, 0.05)$
`corr_100`	`unif1` + Random$(-0.5, 0.5)$
`g1,g2,g3`	Multi-Gaussian(3 peaks, σ=0.05, z=0.75)
`g4,g5,g6`	Multi-Gaussian(50 peaks, σ=0.01, z=0.75)
`z1,z2`	Multi-Zipfian(2500 distinct points, z=0.5)

TABLE I

SYNTHETIC DATA SET USED IN THE EXPERIMENTS.

by a Zipfian distribution with parameter z. In Multi-Zipfian distributions, each dimension has a number of distinct values, and the value sets of each dimension are generated independently. Frequencies are generated according to a Zipfian distribution with parameter z and assigned to randomly chosen cells in the joint frequency distribution matrix.

Workloads: For each experiment we generated a 100-query workload. Each top-k query in the workload uses a scoring function of the form $-\sum_i w_i \cdot (c_i - v_i)^2$ where $0 < w_i \leq 1$ and v_i belongs to c_i's domain, and k ranging from 20 to 100 (values of k in this range are reasonable and were used previously in related work, i.e., [12], [14], [5], [7]). We tried other scoring functions and values of k and obtained similar results, but we omit those for brevity.

Techniques: We compare our proposed strategies of Section III when varying how we approximate the number of iterations of *TA*, and also against existing approaches used in current database systems. Specifically, we consider the following strategies to evaluate top-k queries:

- *Scan*: As a baseline, we consider the scan-based execution plan that modern query optimizers use to answer top-k queries. If a covering index is present, this technique scans the index, calculates the scoring function of each tuple and keeps the best k. If no covering index is present, it uses the existing primary index of the table.

- *TA-Power*: The strategy of Section III that uses a power law to model the number of iterations D. We obtain the best plan using a bottom-up enumeration along with the cost-based pruning

technique of Section III-B.

- *TA-Power-Greedy*: This strategy uses the same cost model as *TA-Power*, but in addition to the cost-based pruning we use a greedy strategy to enumerate alternative plans. The greedy technique is similar to the one used to enumerate index intersection plans in traditional database systems.

- *TA-Linear*: A variant of *TA-Power* that uses linear regression instead of power laws to estimate the number of iterations. We include this technique to illustrate the improvement in accuracy obtained by using power laws.

- *TA-Score*: This strategy does not use the ideas of Section III-D to handle small k values. Instead, when k is smaller than $\frac{N+1}{S+1}$ for a table with N tuples and a sample of size S, we extrapolate s_k, the top-k score as follows. For a scoring function $F(c_1, \ldots, c_m)$ we first obtain the maximum possible score of any tuple, denoted s^{max}, as $F(v_1^{max}, \ldots, v_m^{max})$, where v_i^{max} is the maximum possible value for the attribute score function of attribute c_i. Suppose that $\hat{s_1}$ is the top-score from any tuple in the precomputed sample. We know that there are $\frac{N+1}{S+1}$ expected tuples with scores between $\hat{s_1}$ and s^{max}. We then approximate the top-k score s_k assuming uniformity as $\hat{s_1} + (s^{max} - \hat{s_1}) \cdot (k-1)/(\frac{N+1}{S+1} - 1)$.

For all the sample-based techniques we use a precomputed 1000-tuple sample.

Metrics: We report experimental results using these metrics:

- *Absolute D Error*: When comparing different cost models, we use the absolute D error metric, calculated as follows. For a given technique and query q in the workload, we consider all possible *TA* strategies that are feasible according to the available indexes. For each such strategy s, we estimate the unknown variable D in the cost equation, denoted $D_{est}^{q,s}$ and we also inspect the data to obtain the exact number of iterations $D_{act}^{q,s}$. For a given query q that admits *TA* strategies $s_1, \ldots, s_n$ we calculate the absolute D error as follows:

$$\frac{1}{n} \sum_{s_i} \left| D_{est}^{q,s_i} - D_{act}^{q,s_i} \right|$$

The absolute D error of a workload W is the average absolute D error for all queries in W, and intuitively represents the accuracy of competing models to approximate the execution cost of *TA* strategies.

- *Estimated Execution Time.* After we establish that *TA-Power* results in the most accurate cost estimation among the different *TA* techniques, we conduct an experimental evaluation of its expected performance. For that purpose, we use the following metrics:

- *Scan-Time:* Expected execution time of *Scan*, which is the default execution plan in absence of *TA* strategies.

- *Opt-Time:* Expected execution time taken by the optimal strategy when all information is available. To obtain this value, we inspect the data distribution and obtain the actual number of iterations required by each *TA* strategy. We then calculate the expected cost of each *TA* alternative assuming perfect information and select the most efficient plan among all the *TA* strategies and *Scan*. *Opt-Time* represents the expected time taken by the best possible *TA* or *Scan* alternative when all cost decisions are perfectly accurate, and it is an upper bound of the improvement that we can obtain by adding *TA* strategies to a database system.

- *TA-Time:* Expected time taken by the best plan chosen by the optimizer (either *Scan* or the best *TA* strategy) when using *TA-Power* to estimate the cost of *TA* alternatives. To obtain this value, we calculate the expected cost of each *TA* strategy using *TA-Power* and pick the best plan among the *TA* strategies and *Scan*. Then, we re-evaluate the cost of such best alternative (if it is a *TA* strategy) with the true value of D obtained by inspecting the data. In other words, *TA-Time* is the expected cost using accurate information of the best execution plan chosen using *TA-Power*.

We note that in our experiments we measure expected execution cost (as returned by the query optimizer) rather than actual execution times. We believe that in the context of this work this is a better alternative. In fact, after we estimate the number of iterations D for a given *TA* strategy as shown in Section III-B, our cost model is handled entirely by the optimizer itself. The cost model in the optimizer, however, sometimes results in slight inaccuracies with respect to actual execution times (e.g., sometimes the optimizer might cost a sequential scan slightly cheaper than an index intersection plan when the opposite is actually true). To evaluate our algorithms we assume that the optimizer has a precise model of execution cost, and use its output as a measure of query performance. This way we avoid adding another indirection layer that might ultimately

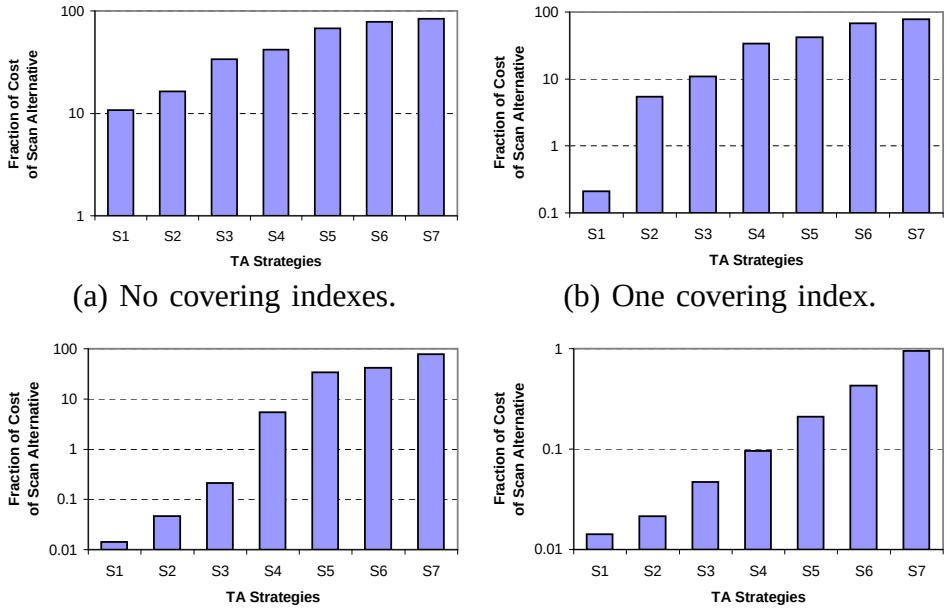
bias our conclusions[8].

A. Evaluating TA Strategies

In this experiment, we issued a 3-dimensional top-100 query over columns `g1`, `g2` and `g3` in the synthetic data set and evaluated the cost of each *TA* strategy (when varying the set of available indexes) by examining the data. Thus, we obtained the accurate cost that competing *TA* strategies would require to evaluate the input query. Figure 6 shows, for different physical configurations, the expected cost of each *TA* strategy compared to that of *Scan* (the figure uses *S1*, *S2*, ..., *S7* to represent the different *TA* strategies in increasing order of expected cost). We can see that when only single-column indexes are present (Figure 6(a)) all *TA* strategies are at least 10 times more expensive than *Scan*. When all covering indexes are available (Figure 6(d)) the situation is reversed and all alternatives are cheaper than *Scan* (the cheapest being almost 2 orders of magnitude faster than *Scan*). Figures 6(b) and 6(c) show results when only one or two covering indexes are available. In those cases, there can be almost four orders of magnitude between the cheapest and most expensive *TA* strategy. This experiment suggests that having an accurate cost model is critical to incorporate *TA* strategies into a relational engine, as bad decisions can dramatically hurt performance. Figure 7 complements this result by comparing the cost of the best *TA* strategy and the corresponding *Scan* strategy for the same query when varying the set of covering indexes that were available (the figure uses *C1*, *C2*, ..., *C8* to represent the optimal *TA* strategy for varying index configurations in increasing order of expected cost). We can see that for some configurations, the best alternative is much better than *Scan*, while for others is over 10 times more expensive.

Analyzing Figure 6(a), it seems that if only single-column indexes were available, *TA* strategies would be significantly worse than *Scan*. We now examine this claim in detail. For that purpose, we use columns `unif` and the different correlated columns `corr_`*x* in the synthetic data set. Consider as an example columns `unif` and `corr_10`. The values in the joint distribution form a narrow band around the diagonal `unif`=`corr_10`. Suppose that we ask a top-100 query with scoring function -(`unif` - $v)^2$ - (`corr_10` - $v)^2$ for some $0 \leq v \leq 1$. The top-100 tuples for

[8]Note, however, that we executed the different execution plans in the database system and in most cases the trends were similar to what the optimizer predicted.



(a) No covering indexes. (b) One covering index.

(c) Two covering indexes. (d) Three covering indexes.

Fig. 6. Estimated execution cost of all *TA* strategies.

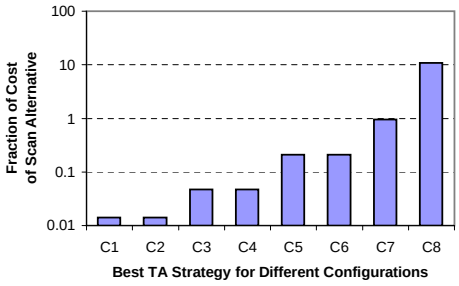


Fig. 7. Estimated execution cost of the best *TA* strategy for varying available indexes.

such a query would lie in a tight circle around (v, v) (see Figure 8). Moreover, the amount of additional tuples that *TA* strategies need to retrieve is minimal (*TA* strategies using indexes over both `unif` and `corr_10` would examine all tuples in the shaded regions of Figure 8). The more correlated the columns, the better the performance of *TA* strategies for top-k queries centered in the diagonal (in fact, the best scenario for a *TA* strategy consists of columns that have the same values).

Figure 9 reports values TA-*Time/Scan-Time* for different workloads, that is, the fraction of the time of *Scan* that we require to evaluate 100 top-k queries when *TA* strategies are additionally available. Again, for this experiment we used the actual values of D to estimate the cost of *TA* strategies to analyze their behavior independently of inaccuracies introduced by *TA-Power*. We

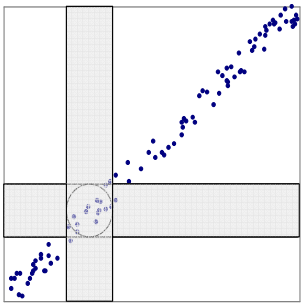
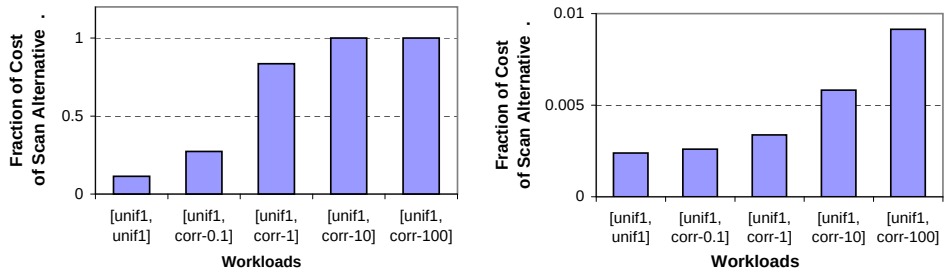


Fig. 8. A top-k query in a correlated data set.

see in Figure 9(a) that when data is very correlated and top-k queries are centered in dense areas, *TA* strategies are more efficient than *Scan*. However, for `Unif1` and `Corr_10` the *Scan* alternative is already better than the best *TA* strategy, and therefore it is always chosen as the optimal plan, resulting in a ratio of one. In contrast, when covering indexes are available (Figure 9(b)) the cost of the best *TA* strategy is always much better than that of *Scan* (as little as one percent of *Scan* in the worst case for this set of experiments). This experiment suggests that *TA* strategies using single-column indexes can be useful in very restricted scenarios. As we will see in the rest of this section, none of the remaining workloads benefits from *TA* when only single-column indexes are available. On the other hand, covering indexes can have an important impact in the time taken by *TA* strategies.



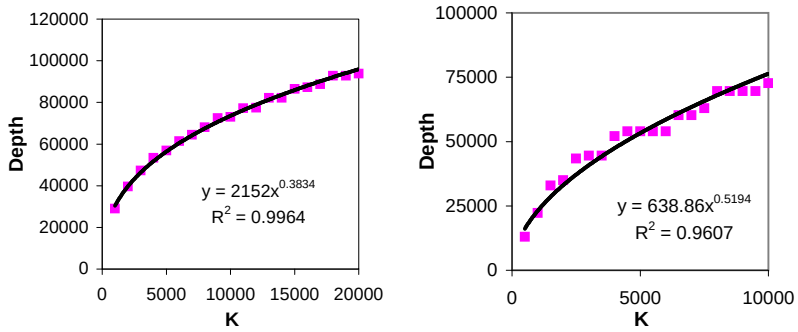
(a) Non-covering indexes. (b) Covering indexes.

Fig. 9. Using covering and non-covering indexes for correlated data.

B. Comparison of Cost Estimators

Having shown that *TA* strategies can be beneficial as alternative execution plans to answer top-k queries, we now analyze the accuracy of different methods that approximate the number of iterations D of *TA* strategies, and therefore their expected costs. We first show the accuracy

of using power laws for approximating the number of iterations of *TA* strategies. As an example, Figure 10 shows two executions of *TA*. We obtained the exact number of iterations for varying k values, and then calculated the best power function that fitted the data. For Figure 10(a) we used the `CovType` data set and a scoring function of the form $\sum_{i=1}^{3} -(c_i - v_i)^2$. For Figure 10(b) we used the two-dimensional Zipfian distribution and the scoring function $w_1 \cdot c_1 + w_2 \cdot c_2$. Again, we note that this is just an approximation as the true underlying function depends on the data distribution. However, as we see in the figures, power laws can be used to approximate the number of iterations with reasonable accuracy.

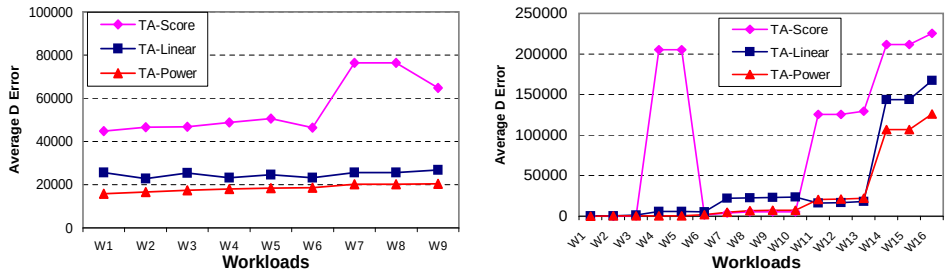


(a) Cover data, 3d query. (b) Zipfian data, 2d query.

Fig. 10. Estimating iterations with power laws.

We next generated several 100-query workloads using different columns from both real and synthetic data sets and obtained the absolute D error for each alternative method. Figure 11 shows the results of this experiment for *TA-Power*, *TA-Linear*, and *TA-Score*. For clarity of presentation, we sorted the workloads by the absolute D error value according to *TA-Power*. Figure 11(a) shows that for the real `CovType` data set, *TA-Power* consistently results in better estimations of D values than both *TA-Linear* and *TA-Score*. Figure 11(b) shows the results corresponding to the synthetic data set. In this case there are larger variations between the accuracy of the different methods, since the data exhibits very different types of correlations. We can see that, although *TA-Power* is not always the most accurate alternative, it is always either the most accurate or very close to it. The other methods, instead, can be significantly less accurate than the best technique for some situations. The reason that *TA-Score* performs significantly worse for some workloads is that it assumes, during extrapolation, that unseen scores are uniformly distributed from the maximum possible score to the top score in the sample, which can be far from true in correlated data sets. *TA-Linear* is based on a principle that is similar to that of *TA-Power*,

and therefore the two techniques perform similarly. However, we can see in the figures that *TA-Linear* is consistently less accurate than *TA-Power*, confirming that the usage of power laws is generally more accurate than using simple linear regression. As explained earlier, *TA-Power* is just a heuristic that approximates D values, but it seems to consistently result in reasonably estimations for a wide spectrum of data and query distributions. We therefore focus exclusively on *TA-Power* in the rest of the work as the method to estimate the cost of *TA* strategies.



(a) `CovType` data set. (b) Synthetic data set.

Fig. 11. Accuracy of different methods to estimate the number of iterations of *TA* strategies.

As explained in Section III-D, we use a "safety factor" α to address the intrinsic limitations in accuracy that are present when estimating the cost of *TA* strategies. Only if the cost of the best *TA* alternative is cheaper than α times the cost of *Scan*, we choose the former over the latter. Figure 12 reports the performance impact of different safety factors. We used a workload consisting of the union of all workloads used in Figure 11(b) over the synthetic data set, and obtained *TA-Time* values for different values of α. We can see that for very small values of α, the *Scan* alternative is almost always chosen and therefore *TA-Time* is closer to *Scan-Time*. On the other extreme, for values of α close to one, inaccuracies in the cost model make the optimizer choose *TA* strategies even though *Scan* would have been better. Somewhere between these two extreme values lies a good tradeoff point that chooses *TA* when it is reasonably certain that it will be more efficient than *Scan*, and the opposite otherwise. We see in the figure that values of α between 0.35 and 0.65 result in the best expected cost. We tried different workloads and the curves for α changed a little but the best expected cost still appeared around $\alpha = 0.5$. We believe that $\alpha = 0.5$ provides a reasonable trade-off and can be used in absence of knowledge about the workload. Otherwise, if performance is crucial, a calibration step similar to what we did in Figure 12 can be conducted with a representative workload to obtain the optimal value

of α. In the remaining of this section we use $\alpha = 0.5$.

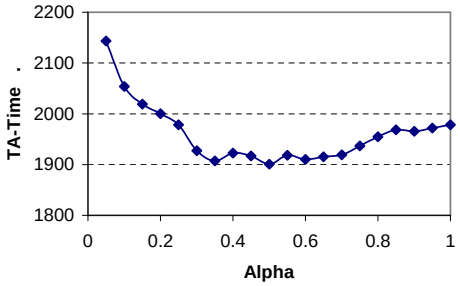
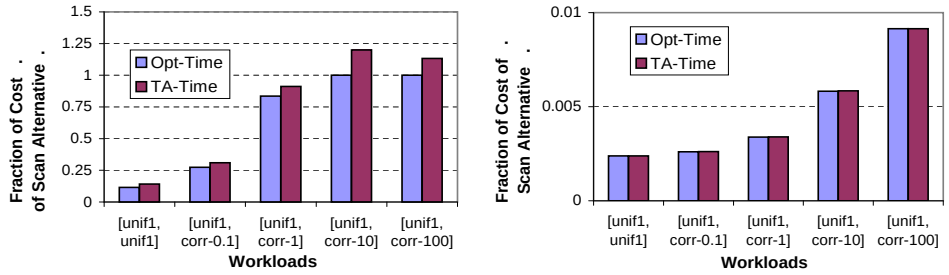


Fig. 12. Determining the safety factor α.

C. Evaluation of TA-Power

In this section we evaluate the expected performance of a database system when *TA* strategies are additionally available to answer top-k queries. Figures 13 and 14 report results for the synthetic data set and Figure 15 does it for the `CovType` real data set.

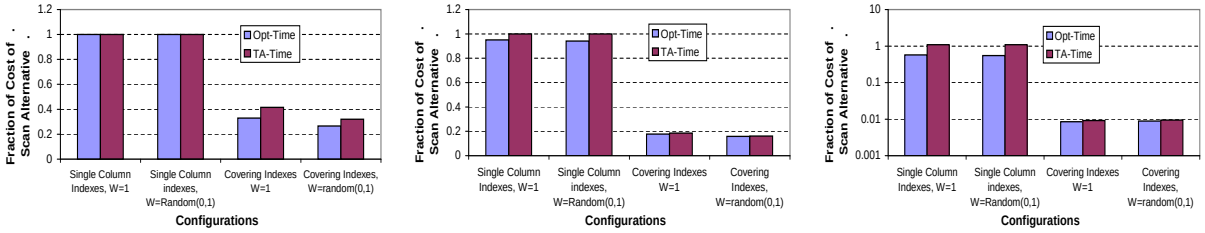


(a) Non-Covering indexes. (b) Covering indexes.

Fig. 13. Using *TA-Power* with correlated data.

We first study the correlated scenarios of Section V-A by re-evaluating the workloads of Figure 9 when *TA-Power* is used to model the cost of *TA* strategies. We can see that when covering indexes are available (Figure 13(b)), *TA-Time* is virtually identical to *Opt-Time* (i.e., *TA-Power* always chooses the best execution plan). Also, note that the expected execution time of the *TA* strategies is over two orders of magnitude faster than that of *Scan*. Since the costs of *TA* strategies in these situations are much more efficient than *Scan*, *TA-Power* almost always chooses the best plan despite inaccuracies in the cost model. On the other hand, when only single-column indexes are available (Figure 13(a)), *TA-Power* misses the best plan more frequently. This is not a big problem for highly correlated data (the first three workloads in Figure 13(a) are still more

efficient than *Scan*). However, for scenarios in which the best strategy is always *Scan*, *TA-Power* can sometimes be chosen instead of *Scan*, thus raising the overall execution time. In fact, the last two workloads in Figure 13(a) represent the worst-case scenario for *TA*. Those workloads do not benefit from *TA* strategies in absence of covering indexes, so any inaccuracy in the cost model can result in worse execution times than when *TA* strategies are unavailable. We note, however, that due to the safety factor, even in those cases the degradation is below 25%. This results reinforce the fact, though, that *TA* strategies offer limited benefit in absence of covering indexes.

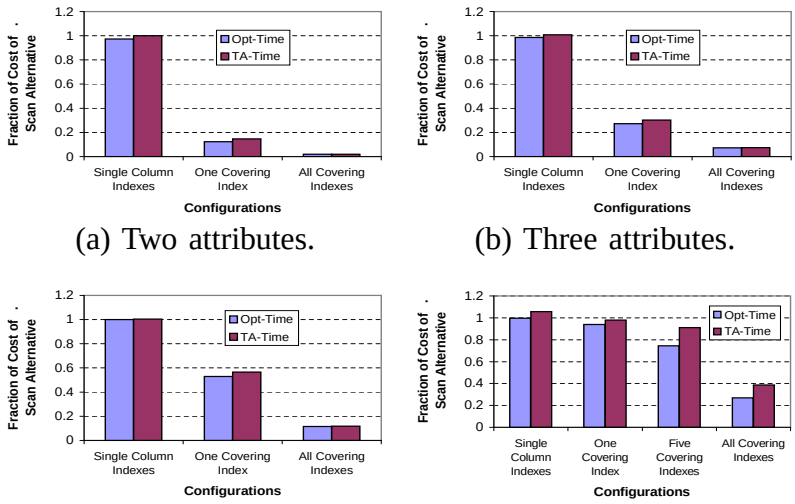


(a) Random data. (b) Gaussian data. (c) Zipfian data.

Fig. 14. Using *TA-Power* with the synthetic data set.

Figure 14 shows results for the synthetic data set. For each workload we tried four different configurations that result from considering (i) single-column versus covering indexes, and (ii) uniform versus random weights in the scoring function $F = -\sum_i w_i(c_i - v_i)^2$. In all cases, *TA-Time* is at most 5% more expensive than *Opt-Time*. When only single-column indexes are available, *TA* strategies are no better than existing *Scan* alternatives (the reason is that the data attributes, while having different degrees of correlation, still require significant iterations for *TA* to terminate). When covering indexes are available, *TA-Time* is from 1% to 40% the value of *Scan*. We also note that when the scoring weights w_i are random, *TA* strategies that exploit covering indexes are slightly more efficient (compared to *Scan*) than when the weights are uniform. The reason is that if weights are random, many times a subset of attributes receives a very small weight and therefore plays almost no role in lowering the threshold value. *TA* strategies that omit using indexes on such attribute require a slightly larger number of iterations over a smaller set of indexes, ultimately saving execution time. For the case of single-column indexes, however, this difference is not enough to make *TA* strategies more efficient than *Scan*.

Finally, Figure 15 shows results for the real `CovType` data set for workloads with varying



(a) Two attributes. (b) Three attributes.

(c) Four attributes. (d) Ten attributes.

Fig. 15. Using *TA-Power* with the `CovType` data set.

number of attributes. For each workload we evaluated three configurations: (i) all single-column indexes are present, (ii) one of the indexes is covering and the rest are single-column, and (iii) all indexes are covering (Figure 15(d) also reports the case with five covering indexes). The results are consistent with those over synthetic data sets, with *TA-Time* being at most 5% more expensive than the optimal strategy (except in Figure 15(d), where *TA-Time* is at most 15% more expensive than the optimal strategy due to the high dimensional nature of the approximation). Specifically, using single-column indexes does not improve performance. In contrast, the more covering indexes are available, the higher the chance that they will be used in some *TA* strategy, decreasing the overall execution time of the workload (when all covering indexes are present, *TA-Time* is below 15% of *Scan-Time* in Figures 15(a-c) and below 40% of *Scan-Time* in Figure 15(d)). We also see that as we increase the number of attributes, the performance of *TA-Time* decreases compared to *Scan*. The reason is that in higher dimensions, the number of iterations required by *TA* to lower the threshold value below the top-k score increases, and so does the expected cost. Figure 15(d) shows that even though it is still possible to use *TA* strategies for up to ten dimensions, the results are less encouraging as in the other cases. Even when five covering indexes are present, *TA* uses over 90% of the time of *Scan*. When all covering indexes are present, *TA* results in 40% of the time of *Scan*, but it requires materializing a large number of redundant structures.

D. Impact of Greedy Search

We re-evaluated all workloads using both *TA-Power* and *TA-Power-Greedy*. In all but three cases, the resulting execution plans were exactly the same for both techniques. For the remaining three cases, *TA-Power-Greedy* resulted in less than 2% degradation with respect to *TA-Power*, by examining a fraction of the space of *TA* strategies.

VI. Conclusions

In this work we studied whether recent algorithms proposed to efficiently answer top-k queries in middleware systems could be adapted to work in the context of a RDBMS. We identified the main challenges in adapting the *TA* algorithm to work inside the relational engine. As with traditional relational processing, the choice of indexes critically determines the execution cost of *TA* strategies. We identified the space of execution alternatives, designed a cost model based on power laws to approximate the cost of each alternative, and provided pruning rules to speed up query optimization. We then showed experimentally that in general, *TA* strategies based on single-column indexes are rarely more efficient than simple sequential scans over the underlying table unless the data is extremely correlated. However, if one or more covering indexes are available, the improvement given by the best *TA* strategy can be of orders of magnitude. We do not expect covering indexes to be built just for answering a given top-k query (in fact, if that were the case, we would be using a full sequential scan to build the index as a prerequisite for a *TA* strategy, which defeats its purpose). In contrast, we believe that for given query workloads, the cost of building a useful set of indexes (once) would be amortized by a large number of queries that subsequently benefit from such indexes. As usual, one needs to be careful since each new index adds overhead for update statements in the workload, but this is the same trade-off that occurs in traditional query processing. We see physical design tuning as an orthogonal problem. Deciding which indexes to create for a given top-k query workload is a very important problem but it lies outside of the scope of this work. Although some of our results seem intuitive, this is the first quantitative evaluation of relying on *TA*-like techniques to answer top-k queries in a RDBMS. Our conclusion is that it is good to incorporate our adapted *TA* strategies to the set of algorithms that a relational optimizer can take advantage of to efficiently evaluate queries.

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